Name ______________________ Class ____________ Date ____________

6-1 Additional Vocabulary Support

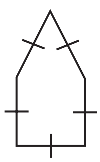
The Polygon Angle-Sum Theorems

Use a word from the list below to complete each sentence.

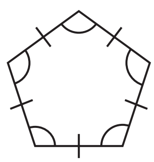
concave	convex	equiangular polygon
equilateral polygon	exterior angle	interior angle
regular polygon		

1. A polygon that has an interior angle greater than 180° is a **concave** polygon.
2. A polygon that has no interior angles greater than 180° is a **convex** polygon.
3. A hexagon in which all angles measure 120° is an example of an **equiangular polygon**.
4. An octagon in which all angles measure 135° and all sides are 6 cm long is an example of a **regular polygon**.
5. An angle inside a polygon is an **interior angle**.

Circle the term that applies to the diagram.

6. 

equiangular　　equilateral　　regular

7. 

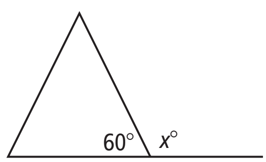
equiangular　　equilateral　　regular

8. 

equiangular　　equilateral　　regular

Multiple Choice

9. What type of angle is the angle labeled $x°$? **B**



Ⓐ acute　　Ⓒ interior

Ⓑ exterior　　Ⓓ straight

10. Which figure is equiangular and equilateral? **I**

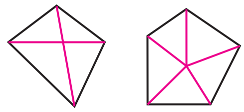
Ⓕ circle　　Ⓖ rectangle　　Ⓗ rhombus　　Ⓘ square

Name ______________________ Class ____________ Date ____________

6-1 Think About a Plan

The Polygon Angle-Sum Theorems

Reasoning Your friend says she has another way to find the sum of the angle measures of a polygon. She picks a point inside the polygon, draws a segment to each vertex, and counts the number of triangles. She multiplies the total by 180, and then subtracts 360 from the product. Does her method work? Explain.



Understanding the Problem

1. According to the Polygon Angle-Sum Theorem, what is the relationship between the number of sides of a polygon and the sum of the measures of the interior angles of a polygon?

The sum is always 180 times two less than the number of sides.

2. How can you write this relationship as an expression in which n is the number of sides? **$(n - 2)180$**

Planning the Solution

3. Mark a point near the center of each figure. Then draw a segment from that point to each vertex as described in the problem.

4. What is the relationship between the number of sides in a polygon and the number of triangles in that polygon?

The number of triangles is the same as the number of sides.

5. What expression can you write to represent multiplying the number of triangles by 180 and then subtracting 360? Let n represent the number of sides. Explain how this expression relates to the picture you drew.

$180n - 360$; the sum of the angles in the triangles is 180 times n, and this is always 360 (the degrees around the point in the center) more than the sum of the angles in the polygon.

Getting an Answer

6. Can you show that the two expressions you wrote are equal? Explain.

Yes; if you apply the Distributive Property, the expressions become the same.

Name ______________________ Class ____________ Date ____________

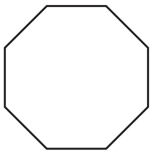
6-1 Practice

Form G

The Polygon Angle-Sum Theorems

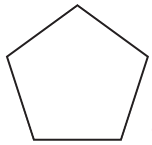
Find the sum of the angle measures of each polygon.

1.



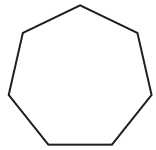
1080

2.



540

3.



900

4. 12-gon **1800**

5. 18-gon **2880**

6. 25-gon **4140**

7. 60-gon **10,440**

8. 102-gon **18,000**

9. 17-gon **2700**

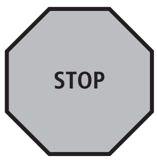
10. 36-gon **6120**

11. 90-gon **15,840**

12. 11-gon **1620**

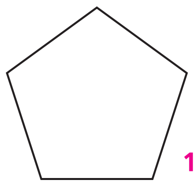
Find the measure of one angle in each regular polygon. Round to the nearest tenth if necessary.

13.



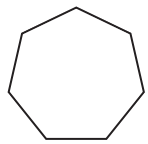
135

14.



108

15.



128.6

16. regular 15-gon **156**

17. regular 11-gon **147.3**

18. regular 13-gon **152.3**

19. regular 24-gon **165**

20. regular 360-gon **179**

21. regular 18-gon **160**

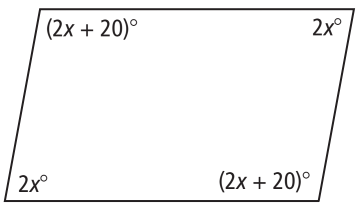
22. regular 36-gon **170**

23. regular 72-gon **175**

24. regular 144-gon **177.5**

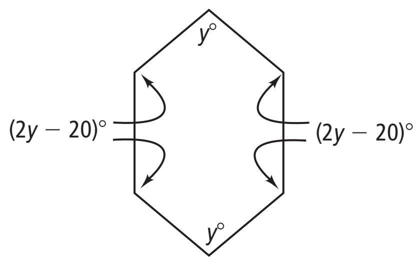
Algebra **Find the missing angle measures.**

25.



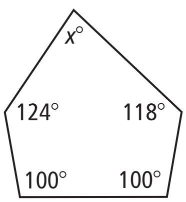
80; 100; 80; 100

26.



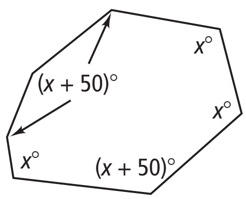
80; 140; 140; 80; 140; 140

27.



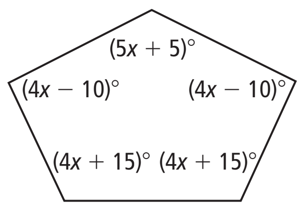
98

28.



100; 100; 150; 100; 150; 150; 150

29.

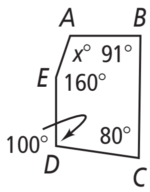


90; 115; 115; 90; 130

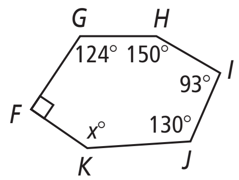
6-1 Practice (continued) *Form G*

The Polygon Angle-Sum Theorems

Algebra **Find the missing angle measures.**

30. 

109

31. 

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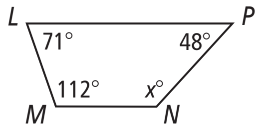
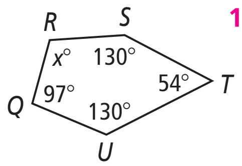
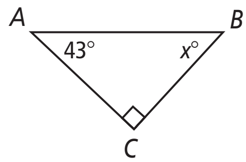
32. 

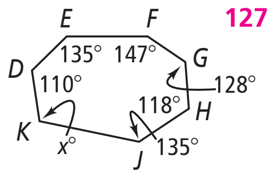
129

33. 

129

34. 

47

35. 

127

Find the measure of an exterior angle of each regular polygon. Round to the nearest tenth if necessary.

36. decagon **36**

37. 16-gon **22.5**

38. hexagon **60**

39. 20-gon **18**

40. 72-gon **5**

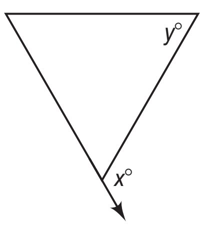
41. square **90**

42. 15-gon **24**

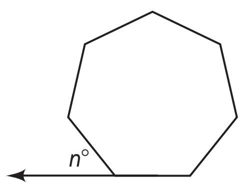
43. 25-gon **14.4**

44. 80-gon **4.5**

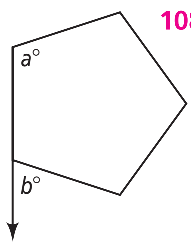
Find the values of the variables for each regular polygon. Round to the nearest tenth if necessary.

45. 

120; 60

46. 

51.4

47. 

108; 72

48. Reasoning Can a quadrilateral have no obtuse angles? Explain.
Yes; a rectangle is a quadrilateral with no obtuse angles.

The measure of an exterior angle of a regular polygon is given. Find the measure of an interior angle. Then find the number of sides.

49. 12 **168; 30**

50. 6 **174; 60**

51. 45 **135; 8**

52. 40 **140; 9**

53. 24 **156; 15**

54. 18 **162; 20**

55. 9 **171; 40**

56. 14.4 **165.6; 25**

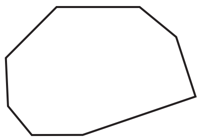
57. 7.2 **172.8; 50**

Name ______________________ Class ____________ Date ____________

6-1 Practice — Form K

The Polygon Angle-Sum Theorems

Find the sum of the angle measures of each polygon.

1. 

To start, determine the sum of the angles using the Polygon Angle-Sum Theorem.

Sum $= (n - 2)180$

$= (\boxed{8} - 2)180 = \boxed{1080}$

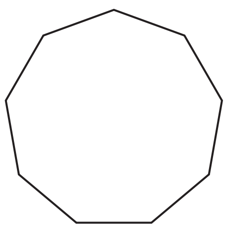
2. 21-gon **3420**

3. 42-gon **7200**

4. 50-gon **8640**

5. 205-gon **36,540**

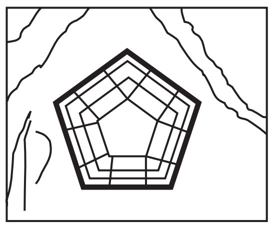
Find the measure of one angle in each regular polygon.

6. 

To start, write the formula used to calculate the measure of an angle of a regular polygon. Then substitute $n = 9$ into the formula.

$$\frac{(n - 2)180}{n} = \frac{(\boxed{9} - 2)180}{\boxed{9}} = \boxed{140}$$

7.  **120**

8.  **108**

Pentagon, Washington, D.C.

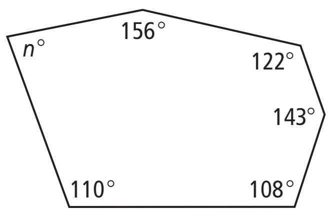
Find the missing angle measures.

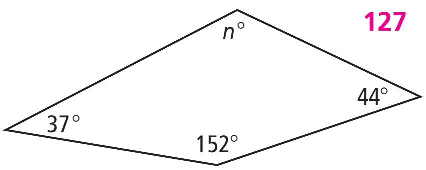
9. To start, determine the sum of the angles using the Polygon Angle-Sum Theorem.

Sum $= (n - 2)180 = (6 - 2)180 = 720$

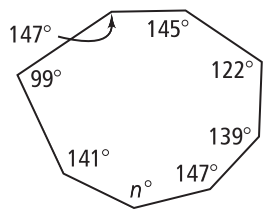
Write an equation relating each interior angle to the sum of the angles.

$n + 156 + 122 + 143 + 108 + 110 = 720$ **81**



10. 

127

11. 

140

Name ______________________ Class ____________ Date ____________

6-1 Practice (continued)

Form K

The Polygon Angle-Sum Theorems

Find the measure of an exterior angle of each regular polygon.

12. 12-gon **30**

13. 24-gon **15**

14. 45-gon **8**

The sum of the angle measures of a polygon with *n* sides is given. Find *n*.

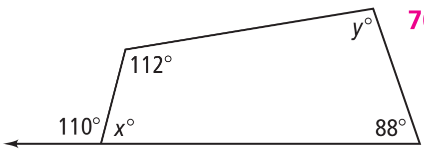
15. 900 **7**

16. 1440 **10**

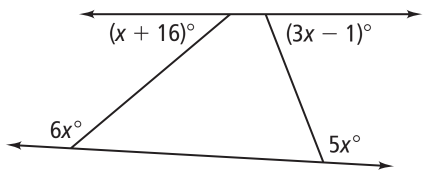
17. 2340 **15**

18. Carly built a Ferris wheel using her construction toys. The frame of the wheel is a regular 16-gon. Find the sum of the angle measures of the Ferris wheel and the measure of one angle. **2520; 157.5**

Algebra **Find the value of each variable.**

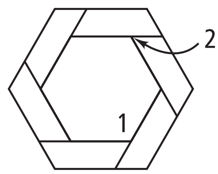
19. 

70; 90

20. 

23

21. Your friend wants to build the picture frame shown at the right.

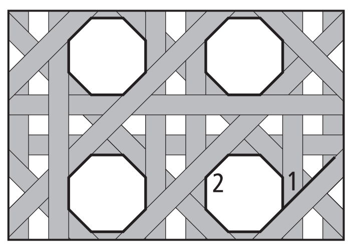


a. What regular polygon is the inside of the frame? **hexagon**

b. Find the measure of each numbered angle. **120; 60**

c. **Reasoning** If you extended one of the exterior sides of the outside of the frame, would the measure of the exterior angle be the same as the measure of $\angle 2$? Explain.
Yes; they are both exterior angles of regular hexagons.

22. Caning chair seats first became popular in England in the 1600s. This method of weaving natural materials produces a pattern that contains several polygons. Identify the outlined polygon. Then, assuming that the polygon is regular, find the measure of each numbered angle. **octagon; 45; 135**



23. **Algebra** The measure of an interior angle of a regular polygon is four times the measure of an exterior angle of the same polygon. What is the name of the polygon? **decagon**

24. **Reasoning** The measure of the exterior angle of a regular polygon is 30. What is the measure of an interior angle of the same polygon? Explain.
150; interior and exterior angles are supplementary

Name ______________________ Class ____________ Date ____________

6-1 Standardized Test Prep

The Polygon Angle-Sum Theorems

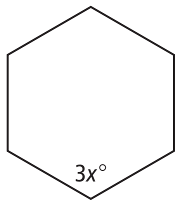
Gridded Response

Solve each exercise and enter your answer on the grid provided.

1. What is the sum of the interior angle measures of a regular octagon?

2. What is the measure of one interior angle of a regular 12-gon?

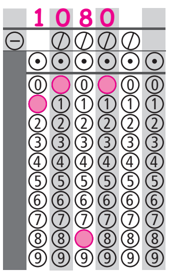
3. What is the value of x in the regular polygon at the right?

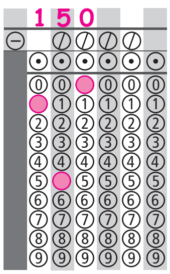


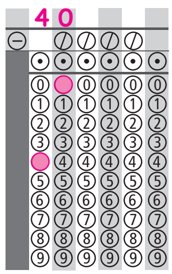
4. What is the measure of an exterior angle of a regular octagon?

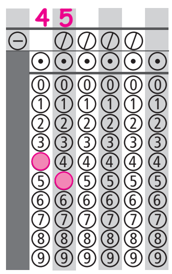
5. If the measure of an exterior angle of a regular polygon is 24, how many sides does the polygon have?

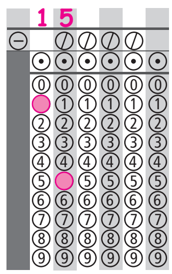
Answers

1.


2.


3.


4.


5.


Name ______________________ Class ____________ Date ____________

6-1 Enrichment

The Polygon Angle-Sum Theorems

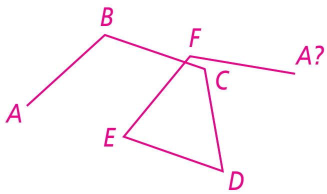
Detective

1. A suspect in a case gives the following information:

"I started at point *A*, then I walked forward 10 paces to *B*. I made a 60° right turn and walked forward 10 more paces to *C*. Then I made a 60° right turn and walked forward yet again 10 more paces to *D*. I stopped to think for a few minutes and decided to make a 120° right turn and walk forward 10 more paces to *E*. I was a bit confused, so I decided to turn right again 110° and walk 10 more paces to *F*. Finally, I made one more right turn of 60° and walked forward 10 paces, and guess what? I was exactly where I started from—*A*!"

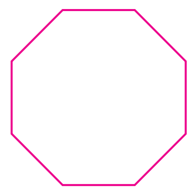
Is the suspect telling the truth? Explain how you know. Draw a sketch of the path of the suspect based on the description.

No; with her first two turns the suspect begins to walk a path that would create a regular hexagon and get her back to her original spot in 5 turns. Each 60° right turn is like turning through the *exterior angle* of the hexagon, creating a 120° interior angle. But when she changes her turns to angles other than 60°, she loses her chance of getting back to *A* as her turns create a sort of spiral.



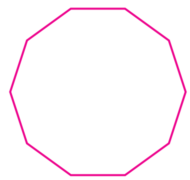
2. Describe a path that would involve eight turns and would result in returning to the same place as the beginning. Draw a sketch and explain how you know the plan works.

Answers may vary. Sample: The angles of a regular octagon are each 135°. If you walk a path with segments of equal length, that exclusively uses 45° turns in the same direction, you will end up back at the starting point.



3. Describe a path that would involve ten turns and would result in returning to the same place as the beginning. Draw a sketch and explain how you know the plan works.

Answers may vary. Sample: The angles of a regular decagon are each 144°. If you walk a path with segments of equal length, that exclusively uses 36° turns in the same direction, you will eventually end up back at the starting point.



4. An equilateral triangle has three 60° angles. Explain why making three 60° left turns does not produce an equilateral triangle.

Turning left 60° is turning through an exterior angle, which results in a 120° interior angle, not the 60° interior angle needed for an equilateral triangle.

Name ______________________ Class ____________ Date ____________

6-1 Reteaching

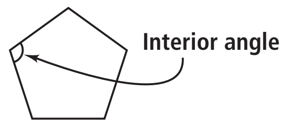
The Polygon Angle-Sum Theorems

Interior Angles of a Polygon

The angles on the inside of a polygon are called *interior angles.*

Polygon Angle-Sum Theorem:

The sum of the measures of the angles of an n-gon is $(n - 2)180$.



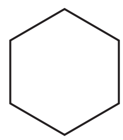
A pentagon has 5 interior angles.

You can write this as a formula. This formula works for regular and irregular polygons.

Sum of angle measures $= (n - 2)180$

Problem

What is the sum of the measures of the angles in a hexagon?



There are six sides, so $n = 6$.

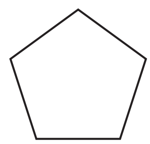
Sum of angle measures	$= (n - 2)180$	
	$= (6 - 2)180$	Substitute 6 for n.
	$= 4(180)$	Subtract.
	$= 720$	Multiply.

The sum of the measures of the angles in a hexagon is 720.

You can use the formula to find the measure of one interior angle of a regular polygon if you know the number of sides.

Problem

What is the measure of each angle in a regular pentagon?



Sum of angle measures	$= (n - 2)180$	
	$= (5 - 2)180$	Substitute 5 for n.
	$= 3(180)$	Subtract.
	$= 540$	Multiply.

Divide by the number of angles:

Measure of each angle	$= 540 \div 5$	
	$= 108$	Divide.

Each angle of a regular pentagon measures 108.

Name ______________________________ Class ____________ Date ____________

6-1 Reteaching (continued)

The Polygon Angle-Sum Theorems

Exercises

Find the sum of the interior angles of each polygon.

1. quadrilateral **360** **2.** octagon **1080** **3.** 18-gon **2880**

4. decagon **1440** **5.** 12-gon **1800** **6.** 28-gon **4680**

Find the measure of an interior angle of each regular polygon. Round to the nearest tenth if necessary.

7. decagon **144** **8.** 12-gon **150** **9.** 16-gon **157.5**

10. 24-gon **165** **11.** 32-gon **168.8** **12.** 90-gon **176**

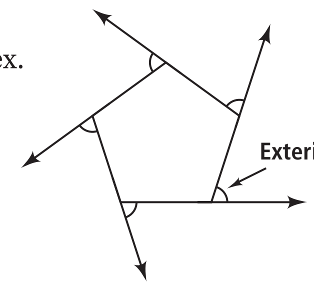
Exterior Angles of a Polygon

The exterior angles of a polygon are those formed by extending sides. There is one exterior angle at each vertex.

Polygon Exterior Angle-Sum Theorem:

The sum of the measures of the exterior angles of a polygon is 360.

A pentagon has five exterior angles. The sum of the measures of the exterior angles is always 360, so each exterior angle of a regular pentagon measures 72.



Exercises

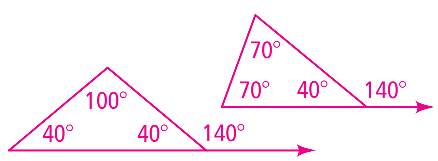
Find the measure of an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

13. octagon **45** **14.** 24-gon **15** **15.** 34-gon **10.6**

16. decagon **36** **17.** heptagon **51.4** **18.** hexagon **60**

19. 30-gon **12** **20.** 28-gon **12.9** **21.** 36-gon **10**

22. Draw a Diagram A triangle has two congruent angles, and an exterior angle that measures 140. Find two possible sets of angle measures for the triangle. Draw a diagram for each. **40, 40, 100; 40, 70, 70**



6-2 Additional Vocabulary Support

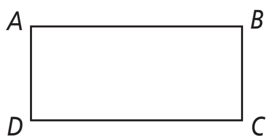
Properties of Parallelograms

Concept List

congruent angles	congruent segments	consecutive angles
diagonal	is parallel to	opposite angles
opposite sides	parallelogram	transversal

Choose the concept from the list above that best represents the item in each box.

1. $\angle A$ and $\angle B$

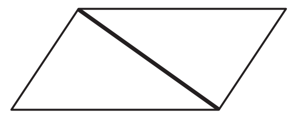


consecutive angles

2. $\parallel$

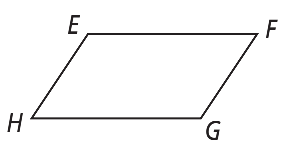
is parallel to

3.



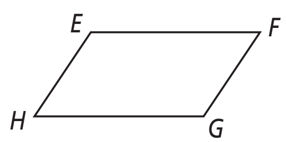
diagonal

4. $\overline{EF}$ and $\overline{HG}$



opposite sides

5. $\angle H$ and $\angle F$



opposite angles

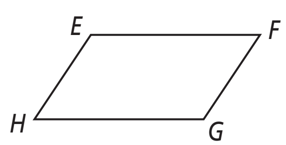
6. $m\angle X = m\angle Y$

congruent angles

7. $WX = YZ$

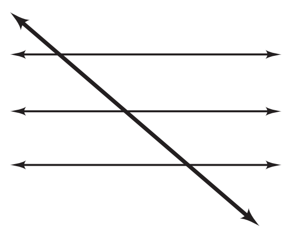
congruent segments

8. $\square EFGH$



parallelogram

9.



transversal

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6-2 Think About a Plan

Properties of Parallelograms

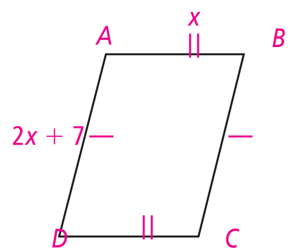
Algebra The perimeter of ▱*ABCD* is 92 cm. *AD* is 7 cm more than twice *AB*. Find the lengths of all four sides of ▱*ABCD*.

1. What is perimeter? **the sum of the measures of the outside edges of the figure**

2. Write a formula to find the perimeter of ▱*ABCD* with side lengths *AB, BC, CD,* and *DA*. $P = AB + BC + CD + DA$

3. What is the relationship between opposite sides of a parallelogram?

They are congruent.



4. Mark the vertices of the parallelogram at the right so it is ▱*ABCD*. Then mark the appropriate sides congruent.

5. How could you use the relationship between opposite sides of a parallelogram to rewrite the formula you wrote in Step 2?

Since opposite sides are congruent, $P = 2AD + 2AB$.

6. Now look back at the relationship between the sides as described in the problem. Let *x* represent *AB*. What expression can you write to represent *AD*? $2x + 7$

7. Write the side lengths from Step 6 on the parallelogram above.

8. How can you use these expressions in the perimeter formula you wrote in Step 5?

Substitute $2x + 7$ **for** *AD*, **and** x **for** *AB*.

9. Rewrite the perimeter formula using the expressions and the value of the perimeter given in the problem. $92 = 2(2x + 7) + 2x$

10. What property can you use to simplify the equation? Rewrite the simplified equation and then solve for *x*.

Distributive Property; $92 = 4x + 14 + 2x$; $x = 13$

11. Now substitute the value of *x* back into the expression to find *AD*. What is the length of each of the four sides of *ABCD*?

$AB = 13$ cm; $BC = 33$ cm; $CD = 13$ cm; $AD = 33$ cm

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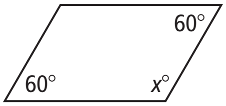
6-2 Practice

Form G

Properties of Parallelograms

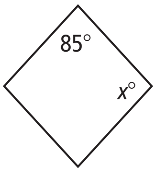
Find the value of *x* in each parallelogram.

1. 

120

2. 

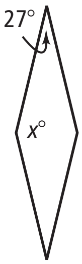
65

3. 

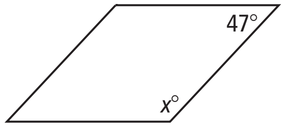
95

4. 

145

5. 

153

6. 

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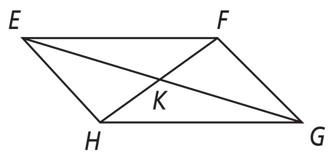
Developing Proof **Complete this two-column proof.**

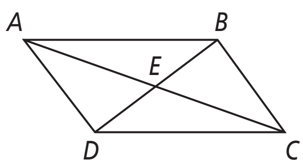


7. Given: ▱$EFGH$, with diagonals $\overline{EG}$ and $\overline{HF}$

Prove: $\triangle EFK \cong \triangle GHK$

Statements	Reasons
1) ? ▱*EFGH*, with diagonals $\overline{EG}$ and $\overline{HF}$	1) Given
2) ? $\overline{FK} \cong \overline{HK}$, $\overline{GK} \cong \overline{EK}$	2) The diagonals of a parallelogram bisect each other.
3) $\overline{EF} \cong \overline{GH}$	3) ? Opposite sides of parallelogram are ≅.
4) ? $\triangle EFK \cong \triangle GHK$	4) ? SSS

Algebra **Find the values for *x* and *y* in ▱*ABCD*.**



8. $AE = 3x$, $EC = y$, $DE = 4x$, $EB = y + 1$ **1; 3**

9. $AE = x + 5$, $EC = y$, $DE = 2x + 3$, $EB = y + 2$ **4; 9**

10. $AE = 3x$, $EC = 2y - 2$, $DE = 5x$, $EB = 2y + 2$ **2; 4**

11. $AE = 2x$, $EC = y + 4$, $DE = x$, $EB = 2y - 1$ **3; 2**

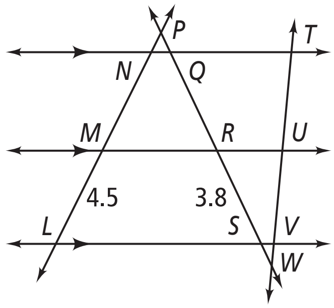
12. $AE = 4x$, $EC = 5y - 2$, $DE = 2x$, $EB = y + 14$ **12; 10**

6-2 Practice (continued)

Form G

Properties of Parallelograms

In the figure, $TU = UV$. Find each length.



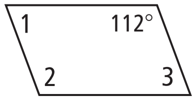
13. NM 4.5 **14.** QR 3.8

15. LN 9 **16.** QS 7.6

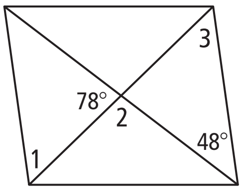
Find the measures of the numbered angles for each parallelogram.

17. 

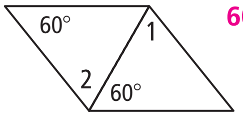
105; 75

18. 

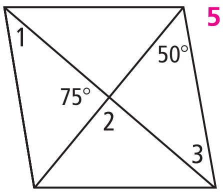
68; 112; 68

19. 

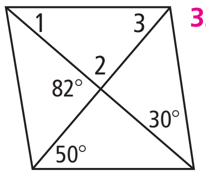
54; 102; 54

20. 

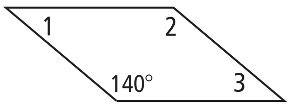
60; 60

21. 

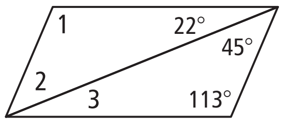
55; 105; 55

22. 

32; 98; 50

23. 

40; 140; 40

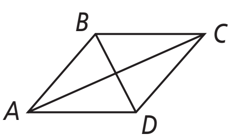
24. 

113; 45; 22

25. Developing Proof A rhombus is a parallelogram with four congruent sides. Write a plan for the following proof that uses SSS and a property of parallelograms.

Given: Rhombus $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E

Prove: $\overline{AC} \perp \overline{BD}$



Use the def. of rhombus, reflexive property, and Theorem 6-6 that states that diagonals of a parallelogram bisect each other to show that two adjacent triangles are congruent by SSS. Use CPCTC to show there is a linear pair of congruent angles, making them right angles, and making the diagonals perpendicular.

6-2 Practice Form K

Properties of Parallelograms

Find the value of *x* in each parallelogram.

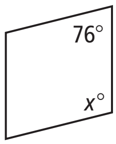
1. 

129

To start, identify the relationship between the marked angles in the diagram.

The marked angles are consecutive angles. By Theorem 6-4, the angles are supplementary.

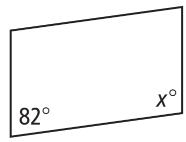
Then write an equation: [**x**] + 51 = 180

2. 

104

3. 

26

4. 

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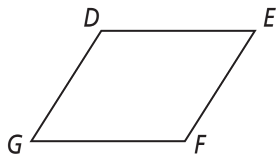
Algebra **Find the values of the variables in ▱*DEFG*.**



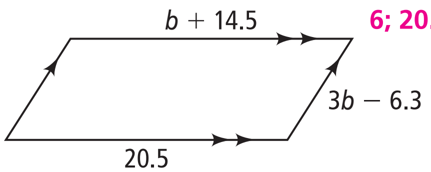
5. $DG = 2x + 2$, $EF = 3x - 3$, $DE = 3x + 1$, $GF = 2x + 6$ **5**

6. $DG = 4a$, $EF = 5a - 6$, $DE = 3a + 2$, $GF = 2a + 8$ **6**

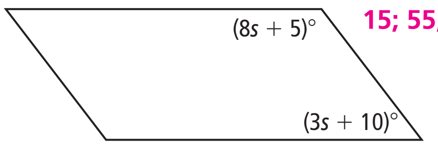
7. $DG = 2r + 3$, $EF = 3r - 3$, $DE = 2r + 6$, $GF = 4r - 6$ **6**

8. $DG = 2t - 10$, $EF = t + 5$, $DE = t + 15$, $GF = 2t$ **15**

Algebra **Find the value of *b* in each parallelogram. Then find each side length or angle measure.**

9. 

6; 20.5; 11.7

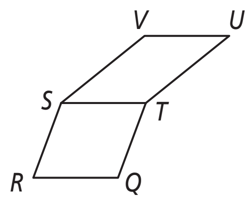
10. 

15; 55; 125

11. Developing Proof Complete this two-column proof.

Given: ▱*QRST*, ▱*TSVU*

Prove: $\overline{RQ} \cong \overline{VU}$



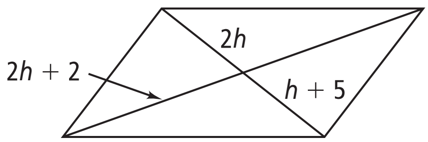
Statements	Reasons
1) ▱*QRST*, ▱*TSVU*	1) __?__ **Given**
2) $\overline{RQ} \cong \overline{ST}$, __?__ **$\overline{ST} \cong \overline{VU}$**	2) Theorem 6-3 Opposite sides of ▱
3) $\overline{RQ} \cong \overline{VU}$	3) __?__ **Trans. Prop. of ≅**

6-2 Practice (continued)

Form K

Properties of Parallelograms

Algebra **Find the value of each variable in each parallelogram.**

12. 

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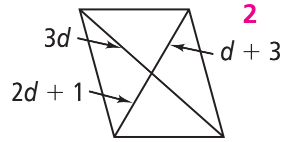
13. 

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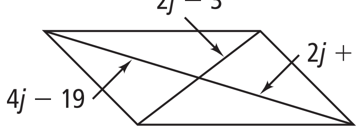
14. 

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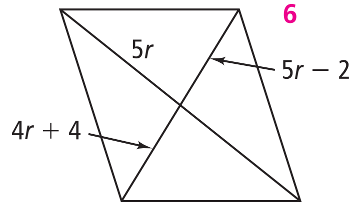
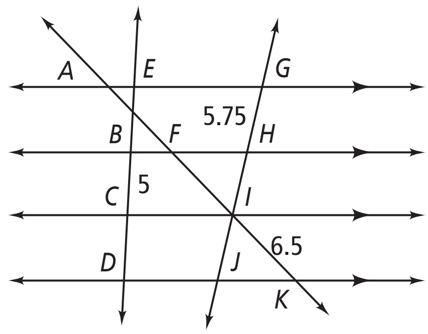
15. 

6

In the figure, $GH = HI = IJ$. Find each length.



16. EB 5

17. BD 10

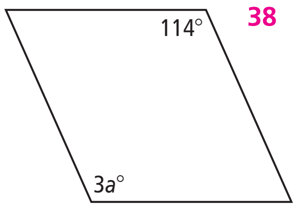
18. AF 6.5

19. AK 19.5

20. CD 5

21. GJ 17.25

Find the value of a in each parallelogram.

22. 

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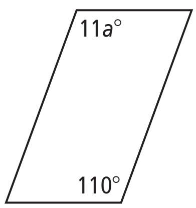
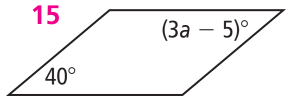
23. 

10

24. 15 

25. The length of one side of a parallelogram is 3 more than twice the length of the adjacent side. The perimeter of the parallelogram is 30 cm. Find the lengths of the two adjacent sides of the parallelogram. **4 cm and 11 cm**

26. Reasoning A classmate draws a parallelogram for which one side is twice as long as the other. If one side is 26 units, what are all the possible lengths of the perimeter? **78 units or 156 units**

6-2 Standardized Test Prep

Properties of Parallelograms

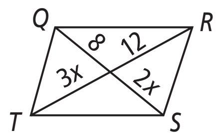
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. In ▱$ABCD$, $m\angle A = 53$. What is $m\angle C$? **B**

Ⓐ 37 Ⓑ 53 Ⓒ 127 Ⓓ 307

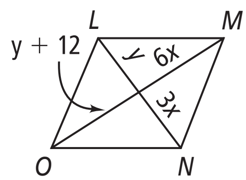
2. What is the value of x in ▱$QRST$? **I**



Ⓕ 16 Ⓗ 8

Ⓖ 12 Ⓘ 4

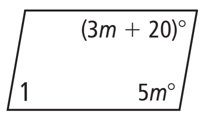
3. What is the value of y in ▱$LMNO$? **C**



Ⓐ 4 Ⓒ 12

Ⓑ 6 Ⓓ 24

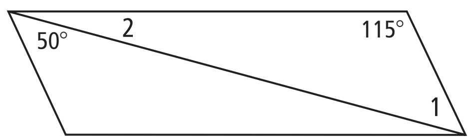
4. What is $m\angle 1$ in this parallelogram? **H**



Ⓕ 20 Ⓗ 80

Ⓖ 60 Ⓘ 100

5. What is $m\angle 2$ in this parallelogram? **C**

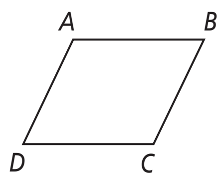


Ⓐ 115 Ⓒ 15

Ⓑ 50 Ⓓ 2

Extended Response

6. Figure $ABCD$ is a parallelogram. What are four geometric attributes you know because $ABCD$ is a parallelogram?



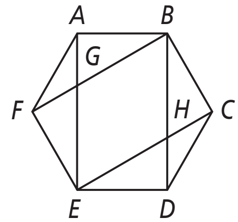
[4] Student lists four attributes from the following list: opposite sides are congruent ($\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$); opposite angles are congruent ($\angle A \cong \angle C$, $\angle B \cong \angle D$); opposite sides are parallel ($\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$); consecutive angles are supplementary ($m\angle A + m\angle B = 180$, $m\angle B + m\angle C = 180$, $m\angle C + m\angle D = 180$, $m\angle D + m\angle A = 180$); the diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other. [3] Student lists three attributes. [2] Student lists two attributes. [1] Student lists one attribute. [0] Student gives no answer or wrong answer based on an inappropriate plan.

Name ______________________ Class ____________ Date ____________

6-2 Enrichment

Properties of Parallelograms

1. In regular hexagon *ABCDEF*, diagonals $\overline{AE}$ and $\overline{BF}$ intersect at *G*. Diagonals $\overline{BD}$ and $\overline{CE}$ intersect at *H*. Prove that quadrilateral *BHEG* is a parallelogram.

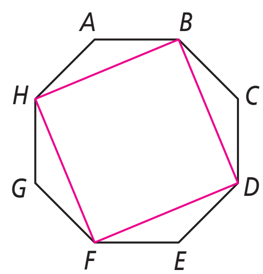


Plan: Prove $\triangle AFB \cong \triangle DEC$ by SAS. Then $\overline{BF} \cong \overline{CE}$. Draw diagonal $\overline{BE}$, and prove $\triangle EFB \cong \triangle BCE$ by SSS. Then $\angle FBE \cong \angle CEB$. Therefore $\overline{BG} \parallel \overline{EH}$. Use a similar argument to prove $\overline{EG} \parallel \overline{BH}$.

Proof:

Statements	Reasons
1. *ABCDEF* is a regular hexagon.	**a.** ? Given
2. $\overline{AF} \cong \overline{DE}$; $\overline{AB} \cong \overline{DC}$, $\angle FAB \cong \angle EDC$	**b.** ? Definition of regular polygon
3. $\triangle AFB \cong \triangle DEC$	**c.** ? SAS
4. $\overline{BF} \cong \overline{CE}$	**d.** ? CPCTC
5. Draw $\overline{BE}$.	**e.** ? Two points determine a segment.
6. $\overline{BE}$ is a diagonal of hexagon *ABCDEF*.	**f.** ? Definition of diagonal
7. $\overline{EF} \cong \overline{BC}$	**g.** ? Definition of regular polygon
8. $\overline{EB} \cong \overline{BE}$	**h.** ? Reflexive Property of Congruence
9. $\triangle EFB \cong \triangle BCE$	**i.** ? SSS
10. $\angle FBE \cong \angle CEB$	**j.** ? CPCTC
11. $\overline{BG} \parallel \overline{EH}$	**k.** ? Converse of Alt. Int. ∠s Thm.
12. $\overline{AF} \cong \overline{DC}$; $\angle AFE \cong \angle BCD$	**l.** ? Definition of regular polygon
13. $\triangle AFE \cong \triangle DCB$	**m.** ? SAS
14. $\overline{AE} \cong \overline{BD}$	**n.** ? CPCTC
15. $\overline{AB} \cong \overline{DE}$	**o.** ? Definition of regular polygon
16. $\triangle ABE \cong \triangle DEB$	**p.** ? SSS
17. $\angle AEB \cong \angle DBE$	**q.** ? CPCTC
18. $\overline{EG} \parallel \overline{BH}$	**r.** ? Converse of Alt. Int. ∠s Thm.
19. *BHEG* is a parallelogram.	**s.** ? Definition of parallelogram

2. How can you make a parallelogram in this regular octagon?



Answers may vary. Sample: Draw diagonals $\overline{BD}$, $\overline{DF}$, $\overline{FH}$, and $\overline{HB}$ as shown. Prove that each small triangle formed is congruent using SAS, then show that the quadrilateral has four congruent sides because corresponding sides of congruent triangles are congruent. Because both pairs of opposite sides of the quadrilateral are congruent, it must be a parallelogram.

Name ______________________ Class ____________ Date ____________

6-2 Reteaching
Properties of Parallelograms

Parallelograms

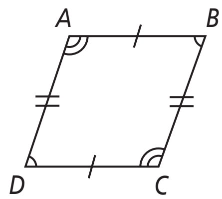
Remember, a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Here are some attributes of a parallelogram:

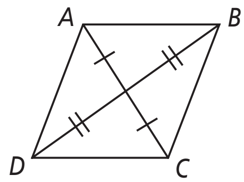
The opposite sides are congruent.

The consecutive angles are supplementary.

The opposite angles are congruent.

The diagonals bisect each other.

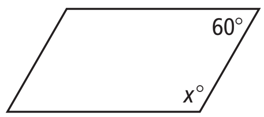




You can use these attributes to solve problems about parallelograms.

Problem

Find the value of x.

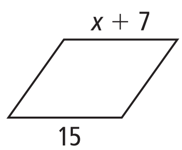


Because the consecutive angles are supplementary,

$$x + 60 = 180$$
$$x = 120$$

Problem

Find the value of x.

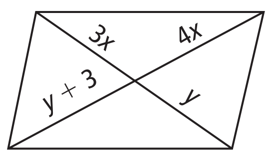


Because opposite sides are congruent,

$$x + 7 = 15$$
$$x = 8$$

Problem

Find the value of x and y.



Because the diagonals bisect each other, $y = 3x$ and $4x = y + 3$.

$4x = y + 3$	
$4x = 3x + 3$	Substitute for y.
$x = 3$	Subtraction Property of =
$y = 3x$	Given
$y = 3(3)$	Substitute for x.
$y = 9$	Simplify.

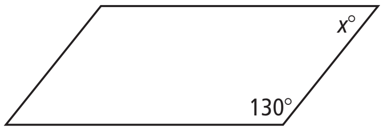
6-2 Reteaching (continued)

Properties of Parallelograms

Exercises

Find the value of *x* in each parallelogram.

1.



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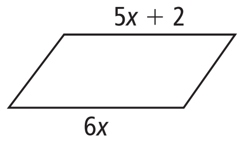
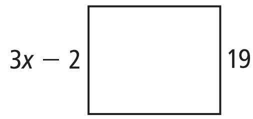
2.



2

3. 35

4.



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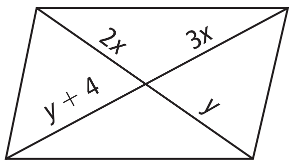
5.



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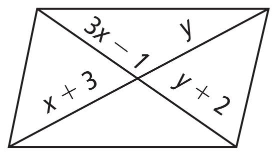

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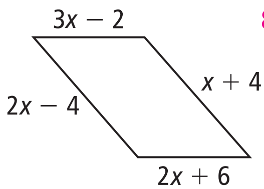

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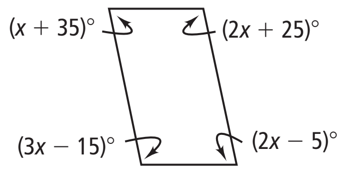

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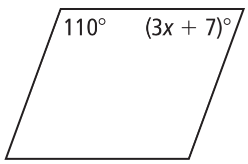
9.



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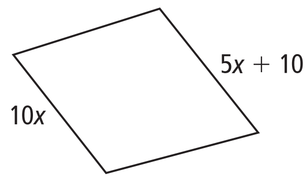
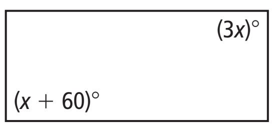
10.



2

11.



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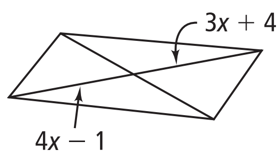
12.



5

13. **Writing** Write a statement about the consecutive angles of a parallelogram.
Consecutive angles of a parallelogram are supplementary.

14. **Writing** Write a statement about the opposite angles of a parallelogram.
Opposite angles of a parallelogram are congruent.

15. **Reasoning** One angle of a parallelogram is 47. What are the measures of the other three angles in the parallelogram?
47, 133, and 133

Name ______________________ Class ____________ Date ____________

6-3 Additional Vocabulary Support

Proving That a Quadrilateral Is a Parallelogram

The **converse** of a statement reverses the conclusion and the hypothesis.	
Example: If A is true, then B is true.	**Converse:** If B is true, then A is true.
Hypothesis: A is true.	Hypothesis: B is true.
Conclusion: B is true.	Conclusion: A is true.
Sample	
Example: If a number is 5 more than 7, then the number is 12.	**Converse:** If a number is 12, then the number is 5 more than 7.

For each statement below, circle the hypothesis and underline the conclusion. Then write the converse.

1. If an apple is red, then the apple is ripe. If an apple is ripe, then it is red.
2. If the tree has leaves, then the season is summer.
   If the season is summer, then the tree has leaves.
3. Complete the converse of this statement: If water is solid, then it is frozen.
   Converse: If water is frozen, then it is solid.

The **converse** of a theorem reverses the conclusion and the hypothesis.	
Sample	
Theorem: If a transversal intersects two parallel lines, then corresponding angles are congruent.	**Converse:** If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

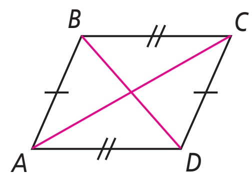
Match each theorem from Section A with its converse in Section B.

Section A:	Section B:
4. If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
5. If a quadrilateral is a parallelogram, then its diagonals bisect each other.	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
6. If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent.	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Name ______________________ Class __________ Date __________

6-3 Think About a Plan

Proving That a Quadrilateral Is a Parallelogram



Prove Theorem 6-8.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$

Prove: *ABCD* is a parallelogram.

1. What is the definition of a parallelogram?

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

2. What are some of the ways that you can prove that two lines are parallel using a transversal?

You can prove that alternate interior, alternate exterior, or corresponding angles are congruent, or that same-side interior angles are supplementary.

3. Draw a transversal on *ABCD* above. This will help you prove that $\overline{BC} \parallel \overline{AD}$.
Check students' work.

4. How can you prove that the triangles formed by the diagonal are congruent?

Because $\overline{BD} \cong \overline{DB}$ by the Reflexive Property, and it is given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $\triangle ABD \cong \triangle CDB$ by SSS.

5. If $\overline{BD}$ is a transversal between lines $\overline{BC}$ and $\overline{AD}$, which angles represent the alternate interior angles? **$\angle CBD$ and $\angle ADB$**

6. How can you prove $\angle CBD \cong \angle ADB$? **CPCTC**

7. What can you conclude about $\overline{BC}$ and $\overline{AD}$?

They are parallel.

8. Draw a transversal to help prove that $\overline{AB} \parallel \overline{DC}$. **Check students' work.**

9. How can you prove that $\angle BAC \cong \angle DCA$?

Because $\overline{CA} \cong \overline{AC}$ by the Reflexive Property, and it is given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, $\triangle ABC \cong \triangle CDA$ by SSS. So $\angle BAC \cong \angle DCA$ by CPCTC.

10. What can you conclude about $\overline{AB}$ and $\overline{CD}$? Why?

They are parallel; $\angle BAC$ and $\angle DCA$ are congruent alternate interior angles.

11. How can you conclude that *ABCD* is a parallelogram?

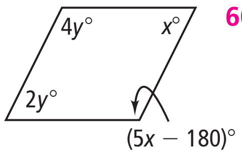
Its opposite sides are parallel, which is the definition of a parallelogram.

6-3 Practice

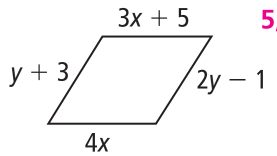
Form G

Proving That a Quadrilateral Is a Parallelogram

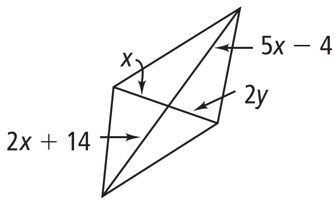
Algebra **For what values of x and y must each figure be a parallelogram?**

1.


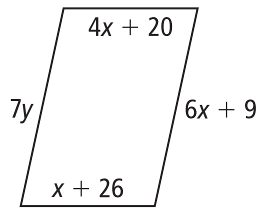
60; 30

2.


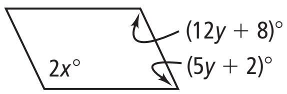
5; 4

3.


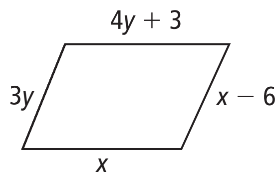
6; 3

4.


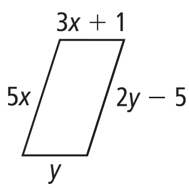
2; 3

5.


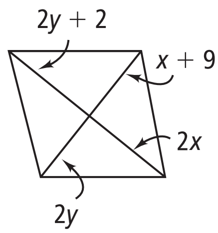
64; 10

6.


15; 3

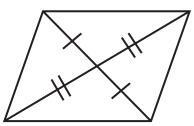
7.


3; 10

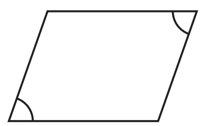
8.


11; 10

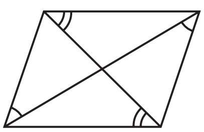
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

9.


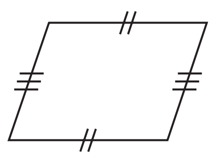
Yes; diagonals bisect each other.

10.


no; not enough information

11.


Yes; two pairs of ≅ alt. int. ∡s implies two pairs of parallel sides.

12.


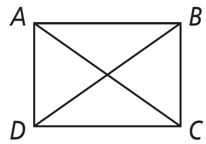
Yes; two pairs of opposite sides are ≅.

Name ______________________________ Class ____________ Date ____________

6-3 Practice (continued) — Form G

Proving That a Quadrilateral Is a Parallelogram

13. Developing Proof Complete the two-column proof. Remember, a rectangle is a parallelogram with four right angles.



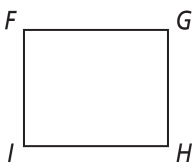
Given: ▱*ABCD*, with $\overline{AC} \cong \overline{BD}$

Prove: *ABCD* is a rectangle.

Statements	Reasons
1) ▱*ABCD*, with $\overline{AC} \cong \overline{BD}$	1) Given
2) ? **$\overline{AD} \cong \overline{BC}$**	2) Opposite sides of a ▱ are congruent.
3) $\overline{DC} \cong \overline{CD}$	3) ? **Reflexive Property**
4) ? **$\triangle ACD \cong \triangle BDC$**	4) SSS
5) $\angle ADC$ and $\angle BCD$ are supplementary.	5) ? **Consecutive ∠s of a ▱ are supp.**
6) $\angle ADC \cong \angle BCD$	6) CPCTC
7) ? **$\angle ADC$ and $\angle BCD$ are right angles.**	7) Congruent supplementary angles are right angles.
8) $\angle DAB$ and $\angle CBA$ are right angles.	8) ? **Opposite ∠s of a ▱ are ≅.**
9) ? ***ABCD* is a rectangle.**	9) Definition of a rectangle

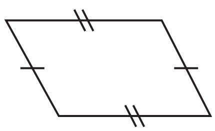
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

14. $\overline{FG} \parallel \overline{IH}$, $\overline{FI} \parallel \overline{GH}$



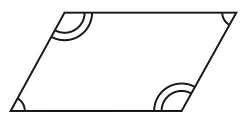
yes; opp. sides parallel

15.



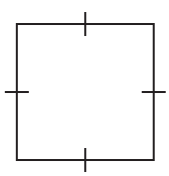
yes; opposite sides ≅

16.



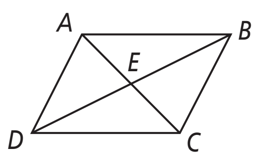
yes; opposite ∠s ≅

17.



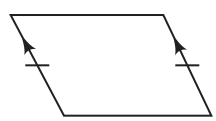
yes; opposite sides ≅

18. $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{ED}$



Yes; diagonals bisect each other.

19.



Yes; one pair of opposite sides is parallel and ≅.

20. Error Analysis It is given that $\overline{MO} \cong \overline{TR}$ and $\overline{NP} \cong \overline{QS}$, where *MNOP* and *TQRS* are parallelograms. A student has said that if those statements are true, then *MNOP* ≅ *TQRS.* Why is this student incorrect?

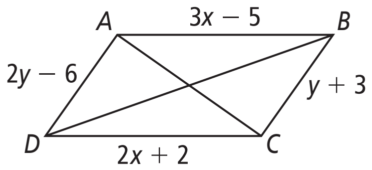
Answers may vary. Sample: It is given that the diagonals are congruent between the two parallelograms, but no angles are given as congruent.

Name ______________________ Class ____________ Date ____________

6-3 Practice *Form K*

Proving That a Quadrilateral Is a Parallelogram

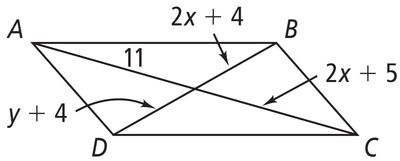
Algebra For what values of *x* and *y* must *ABCD* be a parallelogram?

1. 

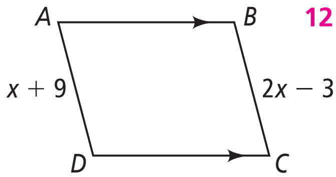
7; 9

To start, write an equation that relates the lengths of opposite sides that have algebraic expressions with the same variable.

$3x - 5 = \underline{\ ?\ }$ **$2x + 2$**

2. 

3; 6

3. 

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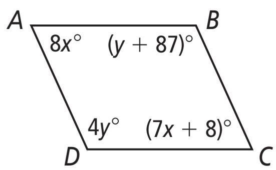
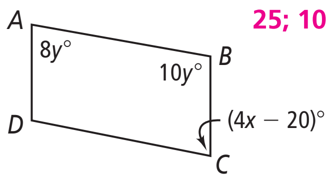
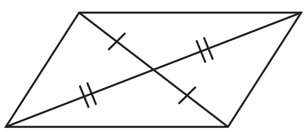
4. 

8; 29

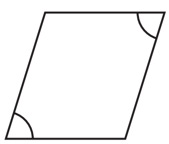
5. 

25; 10

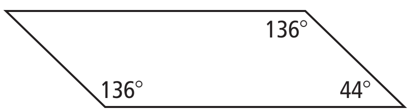
Can you prove the quadrilateral is a parallelogram based on the given information? Explain.

6. 

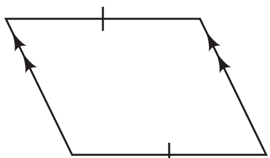
Yes; the diagonals bisect each other.

7. 

No; both sets of opposite angles need to be congruent.

8. 

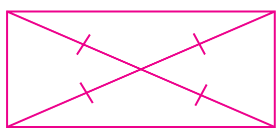
Yes; an angle is supplementary to both of its consecutive angles.

9. 

No; the same pair of sides needs to be both congruent and parallel.

10. Reasoning A classmate drew a quadrilateral with two diagonals. This divided the figure into four isosceles triangles. Is the quadrilateral a parallelogram? Use a drawing to justify your answer.

Yes; since all the sections of the diagonals are congruent, the diagonals bisect each other and the figure is a parallelogram.



6-3 Practice (continued) *Form K*

Proving That a Quadrilateral Is a Parallelogram

11. Developing Proof Complete this two-column proof of Theorem 6-8.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$

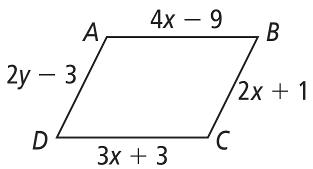
Prove: *ABCD* is a parallelogram.

Statements	Reasons
1) Draw diagonal $\overline{AC}$.	1) Definition of a diagonal
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$	2) _?_ **Given**
3) $\overline{AC} \cong \overline{AC}$	3) _?_ **Reflexive Prop. of ≅**
4) $\triangle ABC \cong$ _?_ **$\triangle CDA$**	4) _?_ **SSS**
5) $\angle B \cong$ _?_ **$\angle D$**	5) _?_ **CPCTC**
6) Draw diagonal $\overline{BD}$.	6) Definition of a diagonal
7) $\overline{BD} \cong \overline{BD}$	7) _?_ **Reflexive Prop. of ≅**
8) $\triangle BCD \cong$ _?_ **$\triangle DAB$**	8) _?_ **SSS**
9) $\angle A \cong$ _?_ **$\angle C$**	9) _?_ **CPCTC**
10) *ABCD* is a parallelogram.	10) _?_ **Theorem 6-10**

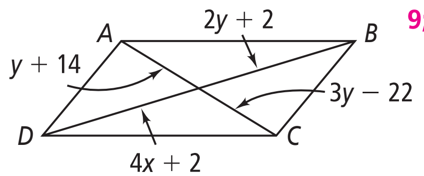
12. Error Analysis A classmate said that a quadrilateral is a parallelogram only if one angle is supplementary to all the others. What is your classmate's error? Explain. **One ∠ must be suppl. to both of its consecutive ∠s. The classmate is describing a rectangle, which is one type of parallelogram.**

For what values of the variables must *ABCD* be a parallelogram?

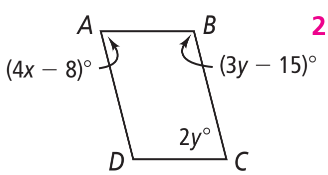
13. **12; 14**



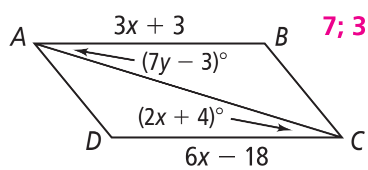
14. **9; 18**



15. **21.5; 39**



16. **7; 3**



Name ______________________ Class ____________ Date ____________

6-3 Standardized Test Prep

Proving That a Quadrilateral Is a Parallelogram

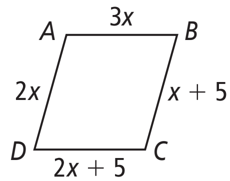
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. For what value of x must *ABCD* be a parallelogram? **A**

Ⓐ 5 Ⓒ 15

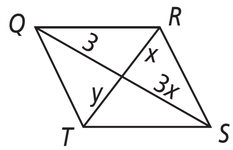
Ⓑ 10 Ⓓ 20



2. For what value of y must *QRST* be a parallelogram? **G**

Ⓕ 0.5 Ⓗ 2

Ⓖ 1 Ⓘ 3



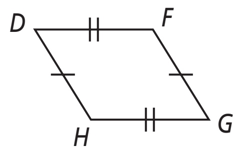
3. Which reason can be used to conclude that *DFGH* is a parallelogram? **C**

Ⓐ There are two pairs of congruent opposite angles.

Ⓑ The diagonals bisect each other.

Ⓒ There are two pairs of congruent opposite sides.

Ⓓ There are two pairs of opposite parallel sides.



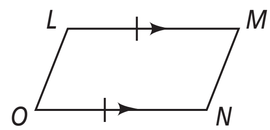
4. Which reason can be used to conclude that *LMNO* is a parallelogram? **I**

Ⓕ There are two pairs of congruent opposite angles.

Ⓖ There are two pairs of congruent opposite sides.

Ⓗ There are two pairs of opposite parallel sides.

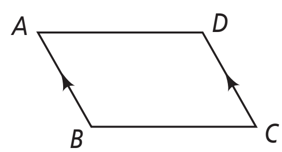
Ⓘ There is one pair of congruent and parallel sides.



Short Response

5. What additional pieces of information could be supplied to make *ABCD* a parallelogram?

[2] Answers may vary. Sample: $\overline{AD} \parallel \overline{BC}$ or $\overline{AB} \cong \overline{DC}$
[1] Only one piece of information is listed. [0] no answer or wrong answer based on an inappropriate plan



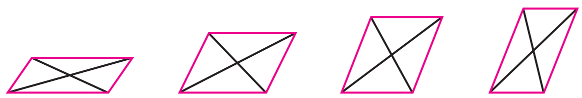
Name ______________________ Class ____________ Date ____________

6-3 Enrichment

Proving That a Quadrilateral Is a Parallelogram

Parallelogram Features

The figures below are diagonals of a parallelogram. Each has one diagonal longer than the other. The pair of diagonals have the same length across all four figures. The diagonals for each figure differ only in the number of degrees between them.



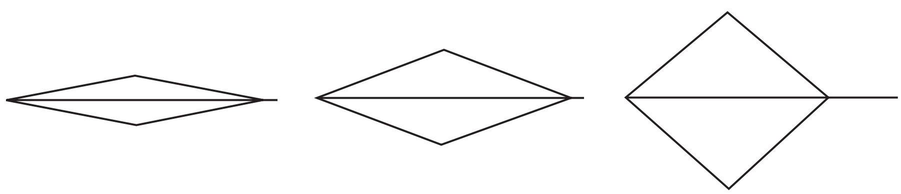
1. Connect the sides on each set of diagonals to form parallelograms. Write any observations you have about the set of parallelograms.
Answers may vary. Sample: They have different heights and bases.

2. The parallelograms are arranged so that their bases all sit on the same line. Draw this line. Also, extend the lines for the side opposite the bases of all the parallelograms that you completed. What do you notice?
Answers may vary. Sample: All these lines are parallel to each other.

3. Extend the lines for all the left and right sides of the parallelograms. What do you notice?
Answers may vary. Sample: The left and right sides of each parallelogram are parallel, but not all sides are parallel to each other.

4. Each of these parallelograms has the same set of diagonals. Name some real-world objects that utilize this feature of parallelograms.
Answers may vary. Sample: lifts, lamp extensions, accordions, jewelry box shelves, clothes drying racks

The parallelograms above have congruent diagonals. The parallelograms below have congruent side lengths. They are like a scissor jack that you would use to lift your car when changing a tire. The horizontal diagonal of a scissor jack is a long screw that, as you turn it, changes the degrees of the angles that it intersects.



5. What is true about the diagonals of the quadrilaterals above?
The diagonals bisect each other, and they are perpendicular.

6. What makes these parallelograms safer for use in a jack design than the shapes at the top of the page? **The perpendicular diagonals would cause a collapsing jack to go straight down instead of to the side.**

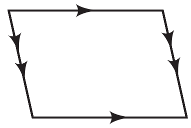
6-3 Reteaching

Proving That a Quadrilateral Is a Parallelogram

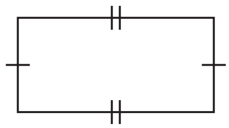
Is a quadrilateral a parallelogram?

There are five ways that you can confirm that a quadrilateral is a parallelogram.

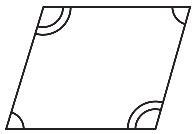
If both pairs of opposite sides are parallel, then the quadrilateral is a parallelogram.



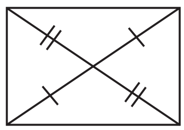
If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.



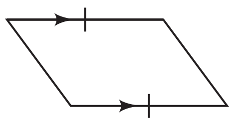
If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.



If the diagonals bisect each other, then the quadrilateral is a parallelogram.

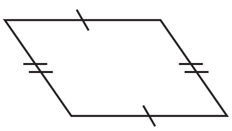


If one pair of sides is both congruent and parallel, then the quadrilateral is a parallelogram.

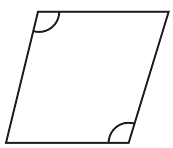


Exercises

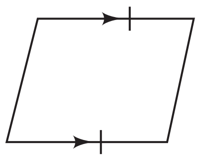
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

1. 

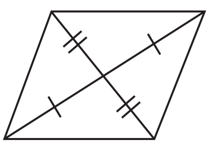
yes; opposite sides ≅

2. 

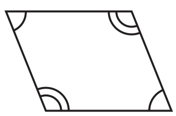
no; not enough info

3. 

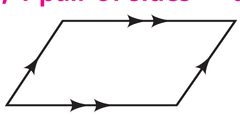
yes; 1 pair of sides ≅ and ∥

4. 

Yes; diagonals bisect each other.

5. 

yes; opposite ∠s ≅

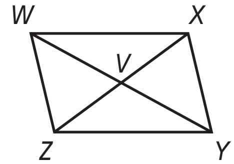
6. 

yes; opposite sides ∥

6-3 Reteaching (continued)

Proving That a Quadrilateral Is a Parallelogram

Determine whether the given information is sufficient to prove that quadrilateral *WXYZ* is a parallelogram.



7. $\overline{WY}$ bisects $\overline{ZX}$ **no**

8. $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \cong \overline{XY}$ **no**

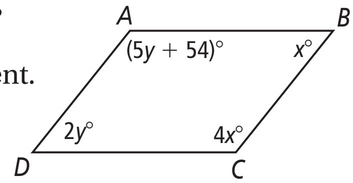
9. $\overline{VZ} \cong \overline{VX}$; $\overline{WX} \cong \overline{YZ}$ **no**

10. $\angle VWZ \cong \angle VYX$; $\overline{WZ} \cong \overline{XY}$ **yes**

You can also use the requirements for a parallelogram to solve problems.

Problem

For what value of *x* and *y* must figure *ABCD* be a parallelogram?



In a parallelogram, the two pairs of opposite angles are congruent. So, in *ABCD*, you know that $x = 2y$ and $5y + 54 = 4x$. You can use these two expressions to solve for *x* and *y*.

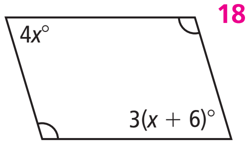
Step 1: Solve for *y*.	$5y + 54 = 4x$	
	$5y + 54 = 4(2y)$	Substitute 2*y* for *x*.
	$5y + 54 = 8y$	Simplify.
	$54 = 3y$	Subtract 5*y* from each side.
	$18 = y$	Divide each side by 3.
Step 2: Solve for *x*.	$x = 2y$	Opposite angles of a parallelogram are congruent.
	$x = 2(18)$	Substitute 18 for *y*.
	$x = 36$	Simplify.

For *ABCD* to be a parallelogram, *x* must be 36 and *y* must be 18.

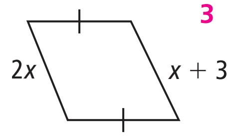
Exercises

For what value of *x* must the quadrilateral be a parallelogram?

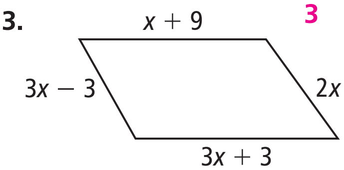
11. **18**



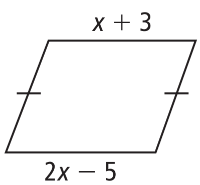
12. **3**



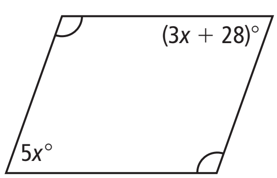
13. **3**



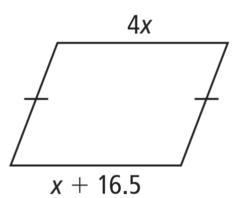
14. **8**



15. **14**



16. **5.5**



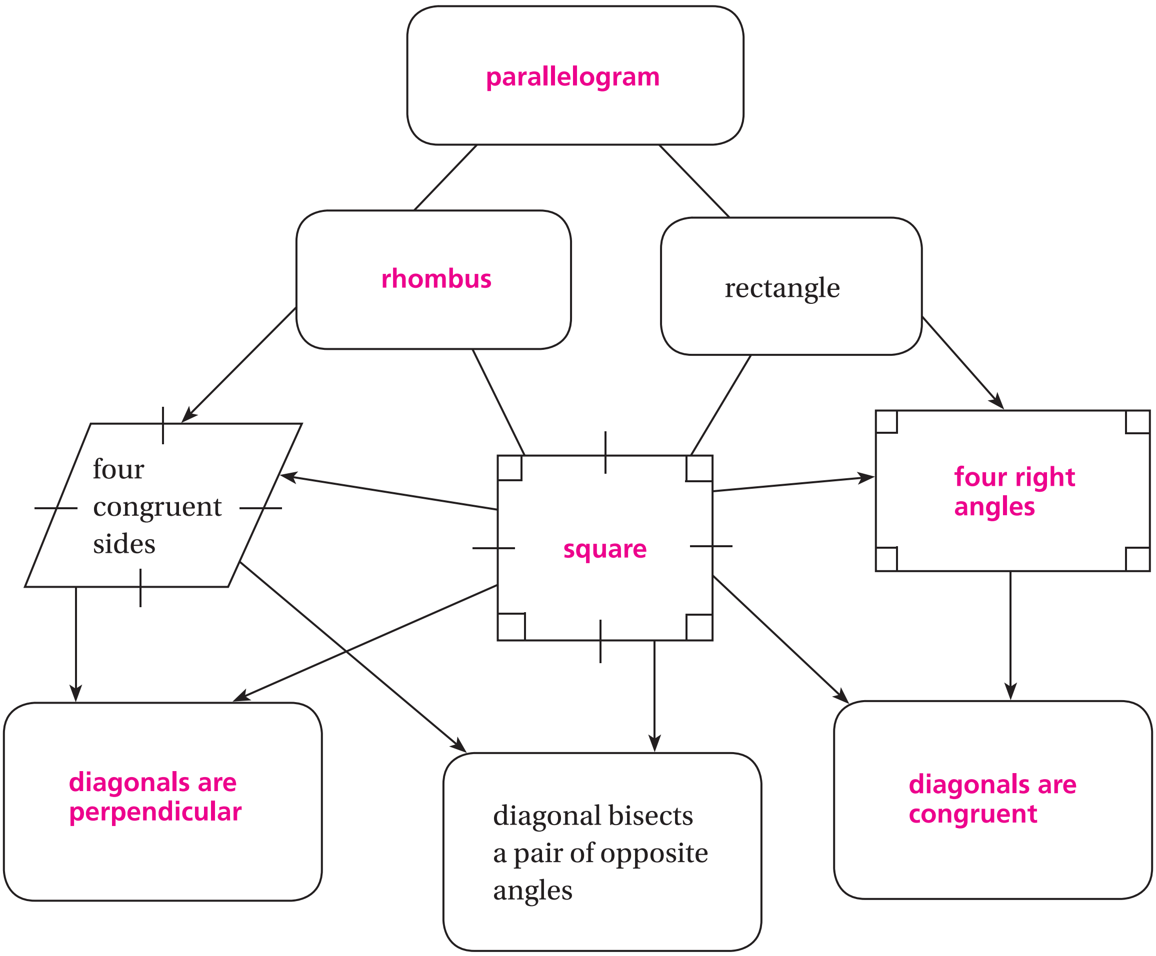
Name ________________ Class ________ Date ________

6-4 Additional Vocabulary Support

Properties of Rhombuses, Rectangles, and Squares

Use the list below to complete the concept map.

diagonals are congruent	diagonals are perpendicular
four right angles	parallelogram
rhombus	square

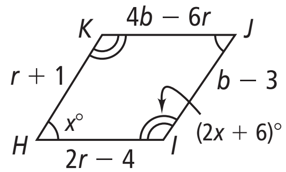


Name ____________________ Class ____________ Date ____________

6-4 Think About a Plan

Properties of Rhombuses, Rectangles, and Squares

Algebra Find the angle measures and the side lengths of the rhombus at the right.



1. What do you know about the sum of the angle measures of a quadrilateral?

The sum is always equal to 360°.

2. Write an equation to represent the sum of the measures of each angle in this rhombus. **$m\angle K + m\angle I + m\angle J + m\angle H = 360$**

3. Based on the diagram, which pairs of angles are congruent? **$\angle K$ and $\angle I$; $\angle J$ and $\angle H$**

4. What is the value of $m\angle H$ given in the diagram? Explain how you can use this value to rewrite the equation from Step 2. Rewrite the equation using this value.

x; replace $m\angle H$ and $m\angle J$ with x because the ∠s are ≅; $m\angle I + m\angle K + x + x = 360$

5. What is the value of $m\angle I$ given in the diagram? Explain how you can use this value to rewrite the equation from Step 4. Rewrite the equation using this value.

$2x + 6$; replace $m\angle K$ and $m\angle I$ with $2x + 6$ because the angles are congruent;

$(2x + 6) + (2x + 6) + x + x = 360$

6. In the space at the right, simplify and solve the equation. **$6x + 12 = 360$; 58**

7. $m\angle H = m\angle J = x =$ **58**

8. $m\angle I = m\angle K = 2x + 6 =$ **122**

9. How can you check that your answer is correct?

The sum of the measures of the four angles should be 360: $58 + 58 + 122 + 122 = 360$.

10. What do you know about the sides of a rhombus? **They are congruent.**

11. What does your answer to Step 10 tell you about the expressions for the sides of the rhombus shown in the diagram? **They are all equal to each other.**

12. Which two expressions in the figure contain the same variable? **$r + 1$ and $2r - 4$**

13. How do these expressions relate to each other? Explain how you can use this relationship to find the value of r. Find the value of r.

They are equal. Set them equal to each other and solve for r; $r + 1 = 2r - 4$; $r = 5$

14. How can you find the length of each side of the rhombus? What is the length of each side? **Substitute 5 into an expression and simplify; 6**

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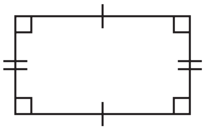
6-4 Practice

Form G

Properties of Rhombuses, Rectangles, and Squares

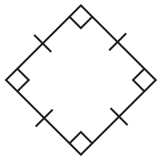
Decide whether the parallelogram is a *rhombus,* a *rectangle,* or a *square.* Explain.

1.



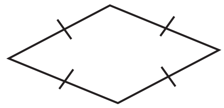
rectangle; 4 rt ∠s, only opposite sides ≅

2.



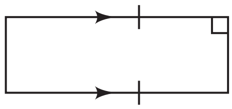
square; 4 right ∠s, 4 ≅ sides

3.



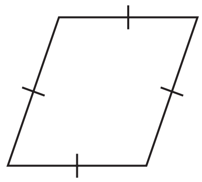
rhombus; no right ∠s, all sides ≅

4.



rectangle; 4 right angles, opposite sides ∥

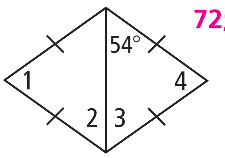
5.



rhombus; 4 ≅ sides, no right angles

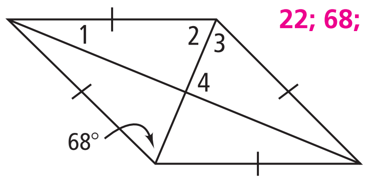
Find the measures of the numbered angles in each rhombus.

6.



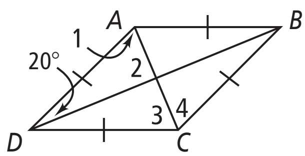
72; 54; 54; 72

7.



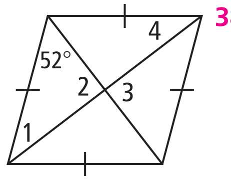
22; 68; 68; 90

8.



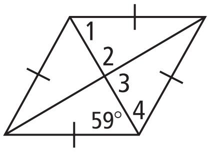
70; 90; 70; 70

9.



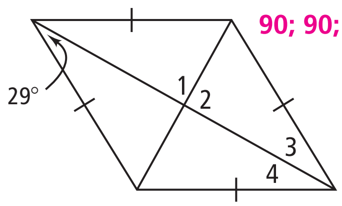
38; 90; 90; 38

10.



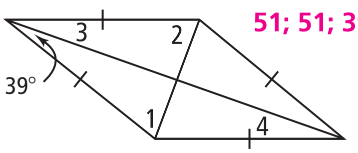
59; 90; 90; 59

11.



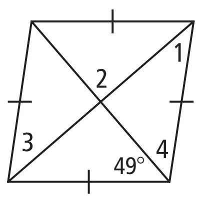
90; 90; 29; 29

12.



51; 51; 39; 39

13.



41; 90; 41; 49

Algebra ***HIJK*** **is a rectangle. Find the value of x and the length of each diagonal.**

14. $HJ = x$ and $IK = 2x - 7$ **7; 7**

15. $HJ = 3x + 5$ and $IK = 5x - 9$ **7; 26**

16. $HJ = 3x + 7$ and $IK = 6x - 11$ **6; 25**

17. $HJ = 19 + 2x$ and $IK = 3x + 22$ **−3; 13**

6-4 Practice (continued) *Form G*

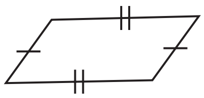
Properties of Rhombuses, Rectangles, and Squares

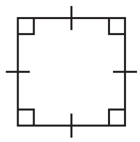
Algebra ***HIJK* is a rectangle. Find the value of *x* and the length of each diagonal.**

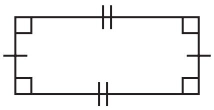
18. $HJ = 4x$ and $IK = 7x - 12$ **4; 16**

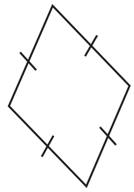
19. $HJ = x + 40$ and $IK = 5x$ **10; 50**

Determine the most precise name for each quadrilateral.

20.  **parallelogram**

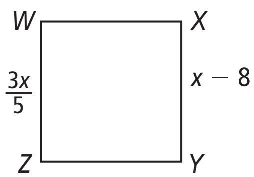
21.  **square**

22.  **rectangle**

23.  **rhombus**

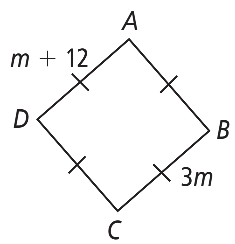
Algebra Find the values of the variables. Then find the side lengths.

24. square *WXYZ*



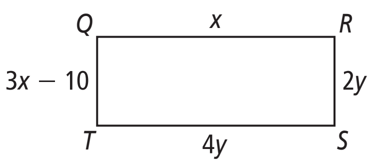
20; 12

25. rhombus *ABCD*



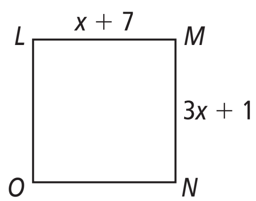
6; 18

26. rectangle *QRST*



4, 1; 4, 2

27. square *LMNO*



3; 10

28. Solve using a paragraph proof.

Given: Rectangle *DVEO* with diagonals $\overline{DE}$ and $\overline{OV}$

Prove: $\triangle OVE \cong \triangle DEV$

Answers may vary. Sample: $\angle OEV$ and $\angle DVE$ are right angles because all angles in a rectangle are right angles. $\overline{VE} \cong \overline{EV}$ by the Reflexive Property. $\overline{OV} \cong \overline{DE}$ because diagonals in a rectangle are congruent. So, $\triangle OVE \cong \triangle DEV$ by HL.

Name ______________________ Class ____________ Date ____________

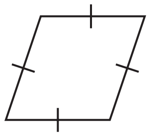
6-4 Practice

Form K

Properties of Rhombuses, Rectangles, and Squares

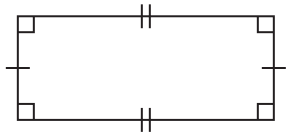
Decide whether the parallelogram is a rhombus, a rectangle, or a square. Explain.

1.



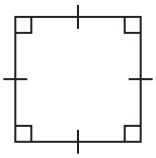
rhombus; four congruent sides, no right angles

2.



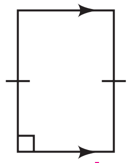
rectangle; opposite sides congruent, four right angles

3.



square; four right angles and four congruent sides

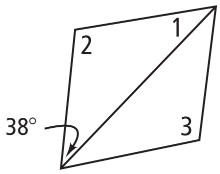
4.



rectangle; right angles, opposite sides congruent

Find the measures of the numbered angles in each rhombus.

5.

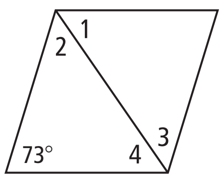


38; 104; 104

To start, a diagonal of a rhombus forms an isosceles triangle with congruent base angles.

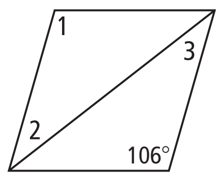
So, $m\angle$ [1] $= 38$.

6.



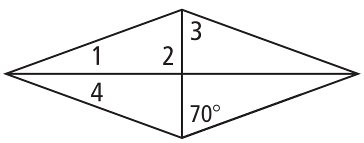
53.5; 53.5; 53.5; 53.5

7.



106; 37; 37

8.

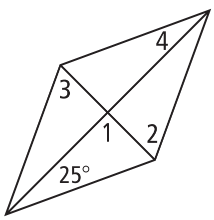


20; 90; 70; 20

To start, the diagonals of a rhombus are perpendicular.

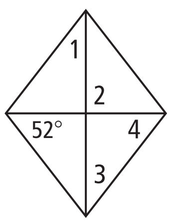
So, $m\angle$ [2] $= 90$.

9.



90; 65; 65; 25

10.



38; 90; 38; 52

Name ______________________ Class ____________ Date ____________

6-4 Practice (continued) — Form K

Properties of Rhombuses, Rectangles, and Squares

Algebra ***QRST* is a rectangle. Find the value of *x* and the length of each diagonal.**

11. $QS = x$ and $RT = 6x - 10$ **2; 2**

To start, write an equation to show the diagonals are congruent.

$\underline{\ ?\ } = \underline{\ ?\ }$ ***x*; 6*x* − 10**

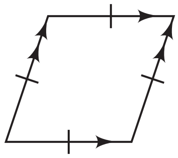
12. $QS = 4x - 7$ and $RT = 2x + 11$
9; 29

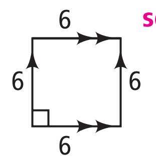
13. $QS = 5x + 12$ and $RT = 6x - 2$
14; 82

14. $QS = 6x - 3$ and $RT = 4x + 19$
11; 63

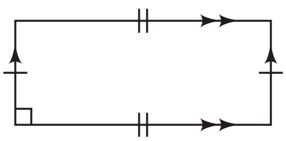
15. $QS = x + 45$ and $RT = 4x - 45$
30; 75

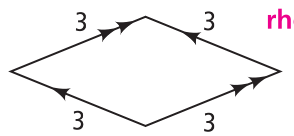
Determine the most precise name for each quadrilateral.

16.  **rhombus**

17. 

square

18.  **rectangle**

19. 

rhombus

Determine whether each statement is *true* or *false*. If it is false, rewrite the sentence to make it true. If it is true, list any other quadrilaterals for which the sentence would be true.

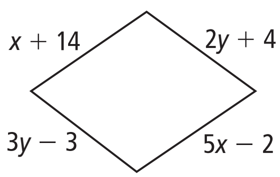
20. Rhombuses have four congruent sides. **true; squares**

21. Rectangles have four congruent angles. **true; squares**

22. The diagonals of a rectangle bisect the opposite angles. **False; the diagonals of a rhombus bisect the opposite angles.**

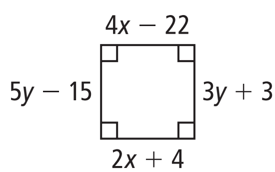
23. The diagonals of a rhombus are always congruent. **False; the diagonals of rectangles or squares are congruent, but not all rhombuses are rectangles or squares.**

Algebra **Find the values of the variables. Then find the side lengths.**

24. 

Rhombus

4; 7; 18

25. 

Square

13; 9; 30

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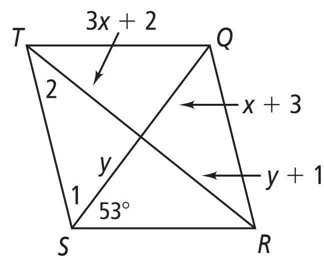
6-4 Standardized Test Prep

Properties of Rhombuses, Rectangles, and Squares

Multiple Choice

For Exercises 1–6, choose the correct letter.

Use rhombus *TQRS* for Exercises 1–4.



1. What is the measure of ∠1? D

Ⓐ 47 Ⓒ 74

Ⓑ 37 Ⓓ 53

2. What is the measure of ∠2? H

Ⓕ 47 Ⓖ 74 Ⓗ 37 Ⓘ 53

3. What is the value of *x*? B

Ⓐ 2 Ⓑ 1 Ⓒ 5 Ⓓ 4

4. What is the value of *y*? F

Ⓕ 4 Ⓖ 3 Ⓗ 2 Ⓘ 1

5. What statement would be sufficient to prove that a quadrilateral is a rhombus? C

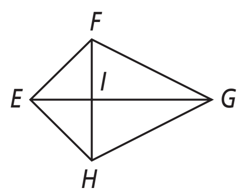
Ⓐ The quadrilateral has four congruent angles.

Ⓑ The quadrilateral has two pairs of parallel sides.

Ⓒ The quadrilateral has four congruent sides.

Ⓓ The quadrilateral has two pairs of congruent angles.

6. *EFGH* is a kite. To prove that the diagonals of a kite are perpendicular, which pair of angles must you prove congruent using CPCTC? H



Ⓕ ∠*EFI* and ∠*EHI* Ⓗ ∠*EIF* and ∠*EIH*

Ⓖ ∠*GFI* and ∠*GHI* Ⓘ ∠*FIE* and ∠*HIG*

Short Response

7. Why is it that the statement "all rhombuses are squares" is false, but the statement "all squares are rhombuses" is true? Explain.

[2] A square has all the properties of a rhombus, plus the additional property of four congruent angles. Not all rhombuses have four congruent angles, so they cannot all be categorized as squares. [1] Student only provides a description of the properties of a square or a rhombus but does not compare the properties of squares and rhombuses. [0] Student does not describe squares and rhombuses or compare their properties.

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6-4 Enrichment

Properties of Rhombuses, Rectangles, and Squares

For Exercises 1–15, write *All, Some,* or *No.* Explain.

1. __?__ rectangles are squares.
   Some; only rectangles with all sides congruent are squares.
2. __?__ isosceles trapezoids are parallelograms.
   No; only one pair of sides is parallel in any trapezoid.
3. __?__ rhombuses are quadrilaterals.
   All; all rhombuses have four sides.
4. __?__ kites are parallelograms.
   No; opposite sides of kites are never congruent.
5. __?__ squares are triangles.
   No; all squares have four sides and all triangles have three sides.
6. __?__ rectangles are regular quadrilaterals.
   Some; squares have four sides of equal length, four right angles, and opposite sides parallel.
7. __?__ quadrilaterals have four congruent angles.
   Some; rectangles are quadrilaterals with four congruent angles.
8. __?__ rectangles are rhombuses.
   Some; only rectangles that have all sides congruent are rhombuses.
9. __?__ trapezoids have one pair of opposite sides parallel.
   All; all trapezoids have only one pair of parallel sides.
10. __?__ trapezoids have two pairs of congruent sides.
    No; if a quadrilateral has two pairs of congruent sides, it is a parallelogram.
11. __?__ kites have two pairs of congruent sides.
    All; all kites have two pairs of adjacent sides congruent.
12. __?__ squares are regular quadrilaterals.
    All; squares are regular quadrilaterals because all their sides and angles are congruent.
13. __?__ kites have congruent diagonals.
    No; only rectangles and isosceles trapezoids have congruent diagonals.
14. __?__ trapezoids have four congruent sides.
    No; trapezoids can have a maximum of three congruent sides.
15. __?__ parallelograms have four congruent angles.
    Some; parallelograms that are rectangles have four congruent angles.

Make up five exercises of your own. Each should be a statement about quadrilaterals starting with a fill-in blank for *All, Some,* or *No.* Write your answers and explanations on a separate sheet of paper. Answers may vary. Sample:

16. __?__ rhombuses are squares.
    Some; rhombuses with four congruent angles are squares.
17. __?__ squares have congruent diagonals.
    All; squares are special rectangles, and all rectangles have congruent diagonals.
18. __?__ trapezoids are isosceles trapezoids.
    Some; trapezoids with congruent legs are isosceles.
19. __?__ squares are parallelograms.
    All; squares have opposite sides parallel.
20. __?__ trapezoids have both pairs of opposite sides parallel.
    No; all trapezoids have only one pair of parallel sides.

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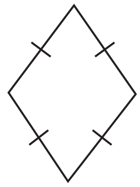
6-4 Reteaching

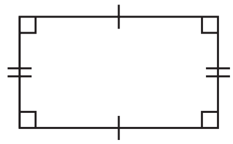
Properties of Rhombuses, Rectangles, and Squares

Rhombuses, rectangles, and *squares* share some characteristics. But they also have some unique features.

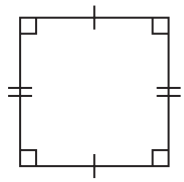
A rhombus is a parallelogram with four congruent sides.



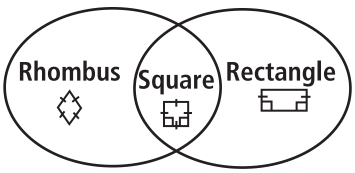
A rectangle is a parallelogram with four congruent angles. These angles are all right angles.



A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus. A square is the only type of rectangle that can also be a rhombus.



Here is a Venn diagram to help you see the relationships.



There are some special features for each type of figure.

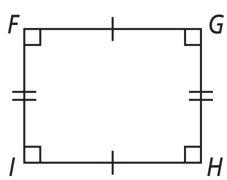
Rhombus: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles.

Rectangles: The diagonals are congruent.

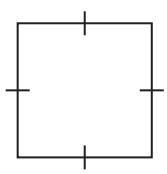
Squares: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles (forming two 45° angles at each vertex).
The diagonals are congruent.

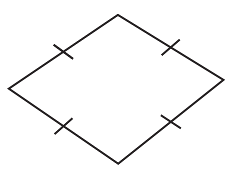
Exercises

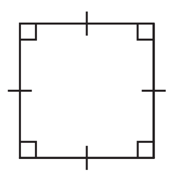
Decide whether the parallelogram is a rhombus, a rectangle, or a square.

1. 

rectangle

2. 
rhombus

3. 
rhombus

4. 
square

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6-4 Reteaching (continued)

Properties of Rhombuses, Rectangles, and Squares

List the quadrilaterals that have the given property. Choose among *parallelogram, rhombus, rectangle,* and *square.*

5. Opposite angles are supplementary.
rectangle, square

6. Consecutive sides are ≅.
rhombus, square

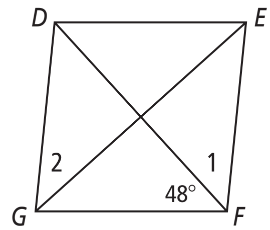
7. Consecutive sides are ⊥.
rectangle, square

8. Consecutive angles are ≅.
rectangle, square

You can use the properties of rhombuses, rectangles, and squares to solve problems.

Problem

Determine the measure of the numbered angles in rhombus *DEFG.*



$\angle 1$ is part of a bisected angle. $m\angle DFG = 48$, so $m\angle 1 = 48$.

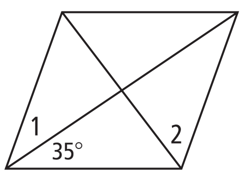
Consecutive angles of a parallelogram are supplementary.
$m\angle EFG = 48 + 48 = 96$, so $m\angle DGF = 180 - 96 = 84$.

The diagonals bisect the vertex angle, so $m\angle 2 = 84 \div 2 = 42$.

Exercises

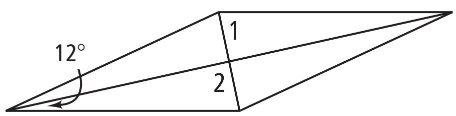
Determine the measure of the numbered angles in each rhombus.

9.



35; 55

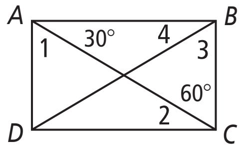
10.



78; 90

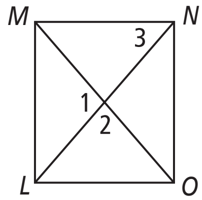
Determine the measure of the numbered angles in each figure.

11. rectangle *ABCD*



60; 30; 60; 30

12. square *LMNO*



90; 90; 45

Algebra ***TUVW* is a rectangle. Find the value of *x* and the length of each diagonal.**

13. $TV = 3x$ and $UW = 5x - 10$
5; 15; 15

14. $TV = 2x - 4$ and $UW = x + 10$
14; 24; 24

15. $TV = 6x + 4$ and $UW = 4x + 8$
2; 16; 16

16. $TV = 7x + 6$ and $UW = 9x - 18$
12; 90; 90

17. $TV = 8x - 2$ and $UW = 5x + 7$
3; 22; 22

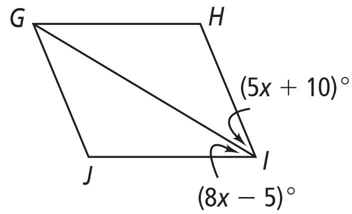
18. $TV = 10x - 4$ and $UW = 3x + 24$
4; 36; 36

Name ______________________ Class ____________ Date ____________

6-5 Additional Vocabulary Support

Conditions for Rhombuses, Rectangles, and Squares

A student wants to find the value for *x*, where ▱*GHIJ* is a rhombus. She wrote the steps to solve the problem on note cards, but they got mixed up.



Think: if ▱GHIJ is a rhombus, the diagonal bisects ∠JIH.

Divide each side by 3:
5 = x.

Subtract 5x from each side:
15 = 3x.

Substitute for m∠GIH and m∠JIG:
5x + 10 = 8x − 5.

Add 5 to each side:
5x + 15 = 8x.

Check:
5(5) + 10 = 8(5) − 5
25 + 10 = 40 − 5
35 = 35

Note that m∠GIH = m∠JIG.

Use the note cards to write the steps in order.

1. First, *think*: If ▱*GHIJ* is a rhombus, the diagonal bisects ∠*JIH*.
2. Second, note that *m*∠*GIH* = *m*∠*JIG*.
3. Third, substitute for *m*∠*GIH* and *m*∠*JIG*: 5*x* + 10 = 8*x* − 5.
4. Fourth, add 5 to each side: 5*x* + 15 = 8*x*.
5. Next, subtract 5*x* from each side: 15 = 3*x*.
6. Then, divide each side by 3: 5 = *x*.
7. Finally, check: 5(5) + 10 = 8(5) − 5; 25 + 10 = 40 − 5; 35 = 35

Name ______________________ Class ____________ Date ____________

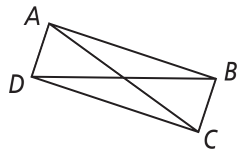
6-5 Think About a Plan

Conditions for Rhombuses, Rectangles, and Squares

Prove Theorem 6-18.

Given: ▱*ABCD*, $\overline{AC} \cong \overline{DB}$

Prove: *ABCD* is a rectangle.



Understanding the Problem

1. What must you prove in order to show that parallelogram *ABCD* is a rectangle?

   All four angles are right angles.

Planning the Solution

2. Using the properties of parallelograms, prove $\triangle BAD \cong \triangle CDA$. Show the steps of this proof below.

Statement	Reason
$\overline{AD} \cong \overline{DA}$	Reflexive Property of Congruence
$\overline{AC} \cong \overline{DB}$	Given
$\overline{AB} \cong \overline{DC}$	Opposite sides of a parallelogram are congruent.
$\triangle BAD \cong \triangle CDA$	SSS Theorem

3. If $\triangle BAD \cong \triangle CDA$, which angles are congruent?

   $\angle BAD \cong \angle CDA$, $\angle DBA \cong \angle ACD$, $\angle ADB \cong \angle DAC$

4. To prove that the angles are right angles you need to use the properties of parallel lines. When a transversal intersects parallel lines, which angles are supplementary?

   same-side interior angles

5. Given that $\overline{AB} \parallel \overline{CD}$, which angles in $\triangle BAD$ and $\triangle CDA$ are supplementary?

   $\angle BAD$ **and** $\angle CDA$

6. If these angles are both congruent and supplementary, what must the measure of each angle be? **90°**

7. How can you prove that the other angles of the parallelogram are right angles?

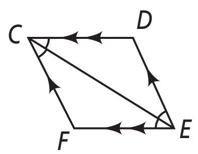
   Opposite angles in a parallelogram are congruent.

Name ______________________ Class ____________ Date ____________

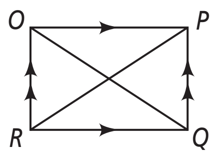
6-5 Practice
Form G

Conditions for Rhombuses, Rectangles, and Squares

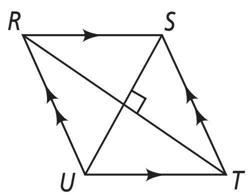
Can you conclude that the parallelogram is a *rhombus,* a *rectangle,* or a *square?* Explain.

1. 

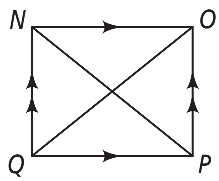
Rhombus; the diagonal bisects opposite angles.

2. 

Parallelogram; opposite sides are parallel, but no other conditions are met.

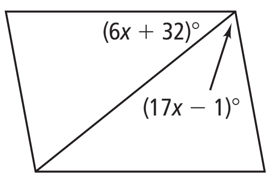
3. 

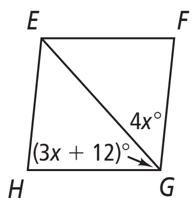
Rhombus; the diagonals are perpendicular.

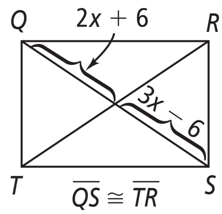
4. $\overline{NP} \cong \overline{OQ}$


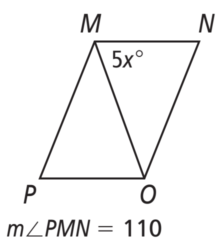
Rectangle; the diagonals are congruent.

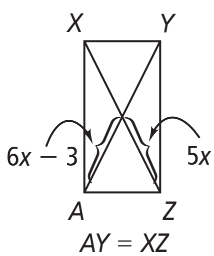
For what value of *x* is the figure the given special parallelogram?

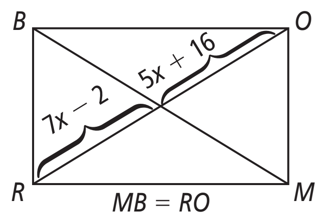
5. rhombus **3**


6. rhombus **12**


7. rectangle **12**


8. rhombus **11**


9. rectangle **3**


10. rectangle **9**


Open-Ended **Given two segments with lengths *x* and *y* ($x \neq y$), what special parallelograms meet the given conditions? Show each sketch.**

11. One diagonal has length *x*, the other has length *y*. The diagonals intersect at right angles. **Rhombus; check students' work.**

12. Both diagonals have length *y* and do not intersect at right angles. **Rectangle; check students' work.**

6-5 Practice (continued) *Form G*

Conditions for Rhombuses, Rectangles, and Squares

For Exercises 13–16, determine whether the parallelogram is a *rhombus,* a *rectangle,* or a *square.* Give the most precise description in each case.

13. A parallelogram has perpendicular diagonals and angle measures of 45, 135, 45, and 135.
rhombus

14. A parallelogram has perpendicular and congruent diagonals.
square

15. A parallelogram has perpendicular diagonals and angle measures that are all 90.
square

16. A parallelogram has congruent diagonals.
rectangle

17. A woman is plotting out a garden bed. She measures the diagonals of the bed and finds that one is 22 ft long and the other is 23 ft long. Could the garden bed be a rectangle? Explain.
No; the diagonals are not congruent.

18. A man is making a square frame. How can he check to make sure the frame is square, using only a tape measure?
He can measure each side to make sure they are equal lengths. Then he can measure the diagonals to make sure that they are also equal lengths.

19. A girl cuts out rectangular pieces of cardboard for a project. She checks to see that they are rectangular by determining if the diagonals are perpendicular. Will this tell her whether a piece is a rectangle? Explain.
No; diagonals are not necessarily perpendicular in rectangles. They must be the same length in rectangles.

20. Reasoning Explain why drawing both diagonals on any rectangle will always result in two pairs of nonoverlapping congruent triangles.
Answers may vary. Sample: In a rectangle the diagonals are congruent, and in any parallelogram, the diagonals bisect each other. The pairs of triangles opposite the center of the rectangle can be shown to be congruent because vertical angles are congruent, and the parts of the diagonals that include each central angle are congruent to corresponding parts in triangles that are opposite the center of the rectangle. So by the SAS Postulate, the triangles are congruent.

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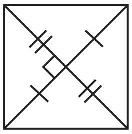
6-5 Practice

Form K

Conditions for Rhombuses, Rectangles, and Squares

Can you conclude that the parallelogram is a rhombus, a rectangle, or a square? Explain.

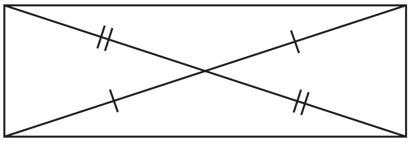
1. 

To start, identify the congruent figures marked in the diagram.

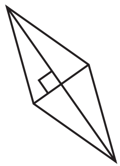
The diagonals bisect each other.

The diagonals intersect at right angles.

Rhombus; the diagonals are perpendicular.

2. 

Neither; the figure could be a ▱ that is neither a rectangle nor a rhombus

3. 

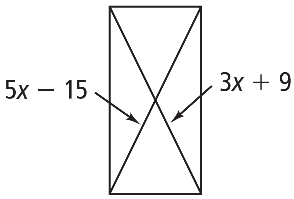
Rhombus; the diagonals are perpendicular.

4. A parallelogram has two pairs of adjacent sides that are congruent. **rhombus**

5. A parallelogram's diagonals form eight congruent angles at the vertices. **square**

Algebra For what value of x is the figure the given special parallelogram?

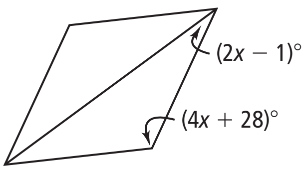
6. rectangle **12**



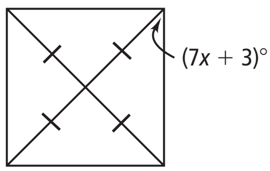
To start, write an equation for the congruent segments.

$\underline{\ ?\ } = \underline{\ ?\ }$ **$5x - 15$; $3x + 9$**

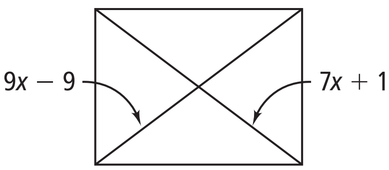
7. rhombus **19.25**



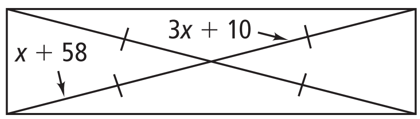
8. square **6**



9. rectangle **5**



10. rectangle **24**



Name ______________________ Class ____________ Date ____________

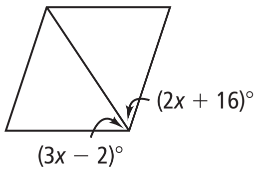
6-5 Practice (continued)

Form K

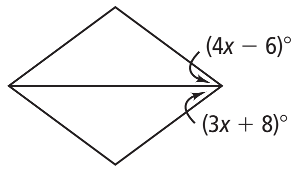
Conditions for Rhombuses, Rectangles, and Squares

Algebra For what value of x is the figure the given special parallelogram?

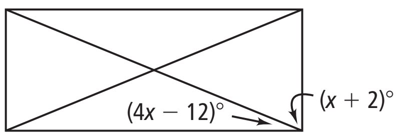
11. rhombus **18**



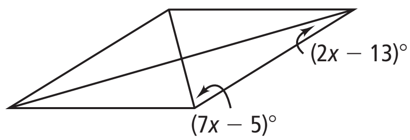
12. rhombus **14**



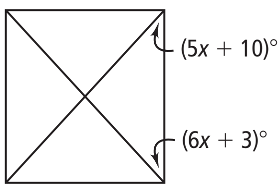
13. rectangle **20**



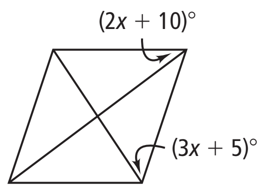
14. rhombus **12**



15. rectangle **7**



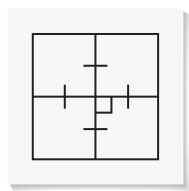
16. rhombus **15**



17. Reasoning Your friend draws a parallelogram with diagonals the length of x and y. Which special type of parallelograms could your friend draw if $x = y$? Which special type of parallelogram could your friend draw if $x \neq y$?

If $x = y$, the figure is definitely a rectangle and possibly a square.
If $x \neq y$, the figure could only be a rhombus.

18. Error Analysis A classmate draws the figure at the right and says that it is a square because its diagonals are both perpendicular and congruent. What is your classmate's error?



The lines drawn are not diagonals so they cannot be used to prove the figure is a square.

19. Students are planning a courtyard garden. They want the garden to be a square. How can the students use ropes to check that the garden is square? Justify your answer and name any theorems you used.

The ropes should be the same length because the diagonals of rectangles are congruent (Theorem 6-18). They should join the ropes at their midpoints because diagonals of parallelograms bisect each other (Theorem 6-11). They need to pull the ropes so they are perpendicular because diagonals of rhombuses are perpendicular (Theorem 6-16). The ends of the ropes will mark the vertices of a square.

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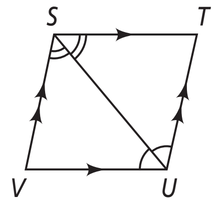
6-5 Standardized Test Prep

Conditions for Rhombuses, Rectangles, and Squares

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which is the most precise name of this figure? **B**



(A) parallelogram (C) rectangle

(B) rhombus (D) square

2. Which of the following conditions or set of conditions must be met for a parallelogram to be a rectangle? **G**

(F) Diagonals are perpendicular.

(G) Diagonals are congruent.

(H) All sides are congruent.

(I) The length of a diagonal is equal to the length of a side.

3. Which of the following conditions or set of conditions is sufficient for a parallelogram to be a square? **A**

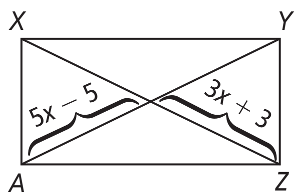
(A) Diagonals are perpendicular and diagonals are congruent.

(B) Diagonals are congruent.

(C) All sides are congruent.

(D) The length of a diagonal is equal to the length of a side.

4. For what value of x is $\square XYZA$ a rectangle? **H**



(F) 2 (H) 4

(G) 3 (I) 5

Short Response

5. The diagonals of a parallelogram are 2.3 cm and 3.2 cm long. Can you tell if the parallelogram is a rhombus? Explain.

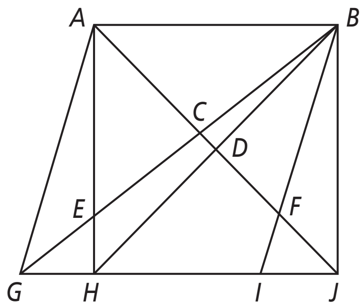
[2] No; no information is given to indicate the diagonals are perpendicular, and no information is given to indicate that a diagonal bisects opposite angles.
[1] Student offers half the explanation. [0] no answer or wrong answer based on an inappropriate plan

Name ______________________________ Class ____________ Date ____________

6-5 Enrichment

Conditions for Rhombuses, Rectangles, and Squares

Special Parallelograms in Patterns



A quilter is designing a quilt. It is to have a repeating pattern, the basic unit of which is shown in the diagram. Points where two or more pieces of cloth come together are labeled.

In the pattern:

$\triangle AGH \cong \triangle BIJ$, $AB = HJ$, $AJ = HB$, $\overline{AB} \parallel \overline{GI}$, $m\angle ABC = m\angle CBD + m\angle DBF$, $m\angle AGB = m\angle JGB$

1. a. Name a rectangle in the pattern. **▱*ABJH***

b. How do you know this is a rectangle?

Because it is given that $\triangle AGH \cong \triangle BIJ$, then $\overline{AH} \cong \overline{BJ}$ by CPCTC. It is given that $AB = HJ$, so $\overline{AB} \cong \overline{HJ}$ by the definition of congruence. So, *ABJH* is a parallelogram, because both pairs of opposite sides are congruent. It is also given that $AJ = HB$, so *ABJH* is a rectangle because it is a parallelogram with congruent diagonals.

2. a. Name a non-rectangular rhombus in the pattern. **▱*ABIG***

b. Prove that this figure is a rhombus.

Statements	Reasons
1) $\triangle AGH \cong \triangle BIJ$	1) Given
2) $\overline{AG} \cong \overline{BI}$, $\overline{GH} \cong \overline{IJ}$	2) CPCTC
3) $AG = BI$, $GH = IJ$	3) Definition of congruence
4) $\overline{AB} \parallel \overline{GI}$, $AB = HJ$	4) Given
5) $HJ = HI + IJ$, $GI = HI + GH$	5) Segment Addition Postulate
6) $HJ = HI + GH$	6) Substitution Property
7) $HJ = GI$	7) Substitution Property
8) $AB = GI$	8) Transitive Property of Equality
9) *ABIG* is a parallelogram.	9) One pair of opp. sides ≅ and parallel
10) $m\angle AGB = m\angle JGB$	10) Given
11) $m\angle ABC = m\angle CBD + m\angle DBF$	11) Given
12) $m\angle ABC = m\angle CBF$	12) Angle Addition Postulate
13) *ABIG* is a rhombus.	13) Parallelogram with a diagonal that bisects opposite angles.

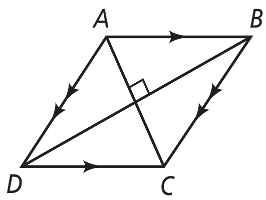
3. Design your own geometric pattern with rhombuses, squares, and rectangles. Write a description of the relationship among line segments and angles in your drawing. **Check students' work.**

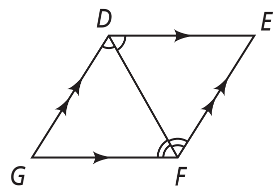
Name ______________________ Class ____________ Date ____________

6-5 Reteaching

Conditions for Rhombuses, Rectangles, and Squares

A parallelogram is a rhombus if either of these conditions is met:



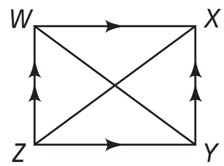


1) The diagonals of the parallelogram are perpendicular. (Theorem 6-16)

2) A diagonal of the parallelogram bisects a pair of opposite angles. (Theorem 6-17)

A parallelogram is a rectangle if the diagonals of the parallelogram are congruent.

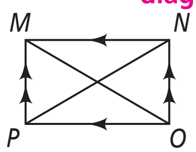
$\overline{WY} \cong \overline{XZ}$



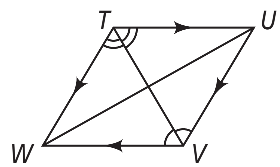
Exercises

Classify each of the following parallelograms as a *rhombus,* a *rectangle,* or a *square.* For each, explain.

1. $\overline{MO} \cong \overline{PN}$ **Rectangle; the diagonals are ≅.**

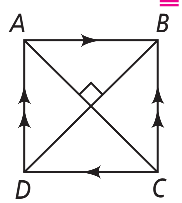


2.



Rhombus; the diagonals bisect opposite angles.

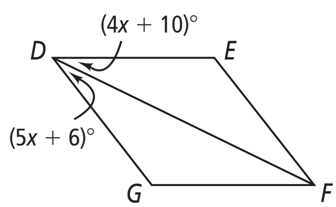
3. $\overline{AC} \cong \overline{BD}$ **Square; the diagonals are ≅ and ⊥.**



Use the properties of rhombuses and rectangles to solve problems.

Problem

For what value of x is ▱$DEFG$ a rhombus?



In a rhombus, diagonals bisect opposite angles.

So, $m\angle GDF = m\angle EDF$.

$(4x + 10) = (5x + 6)$	Set angle measures equal to each other.
$10 = x + 6$	Subtract 4x from each side.
$4 = x$	Subtract 6 from each side.

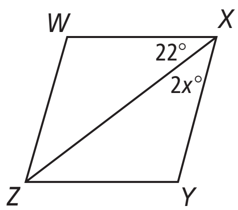
Name ______________________ Class ____________ Date ____________

6-5 Reteaching (continued)

Conditions for Rhombuses, Rectangles, and Squares

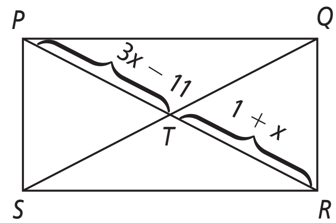
Exercises

4. For what value of x is $\square WXYZ$ a rhombus? **11**

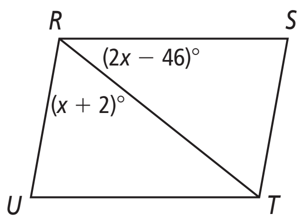


5. $SQ = 14$. For what value of x is $\square PQRS$ a rectangle?

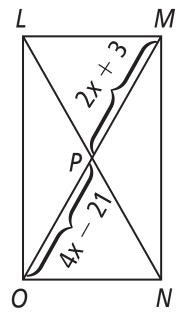
Solve for PT. Solve for PR. **6; 7; 14**



6. For what value of x is $\square RSTU$ a rhombus? What is $m\angle SRT$? What is $m\angle URS$? **48; 50; 100**



7. $LN = 54$. For what value of x is $\square LMNO$ a rectangle? **12**



8. Given: $\square ABCD$, $\overline{AC} \perp \overline{BD}$ at E.

Prove: $ABCD$ is a rhombus.

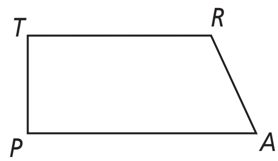
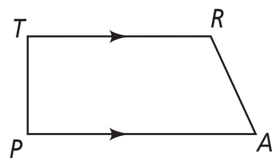
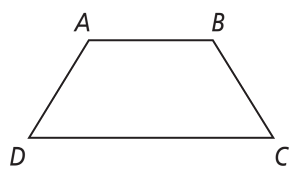
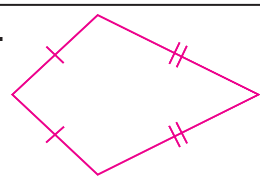
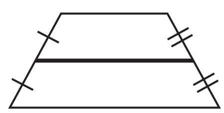
Statements	Reasons
1) $\overline{AE} \cong \overline{CE}$	1) ? **Diagonals of a ▱ bisect each other.**
2) $\overline{AC} \perp \overline{BD}$ at E	2) ? **Given**
3) ? **∠AED and ∠CED are right angles.**	3) Definition of perpendicular lines
4) ? **∠AED ≅ ∠CED**	4) ? **All right angles are congruent.**
5) ? **$\overline{DE} \cong \overline{DE}$**	5) Reflexive Property of Congruence
6) $\triangle AED \cong \triangle CED$	6) ? **SAS Postulate**
7) $\overline{AD} \cong \overline{CD}$	7) ? **CPCTC**
8) ? **$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$**	8) Opposite sides of a ▱ are ≅.
9) ? **$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$**	9) ? **Transitive Property of Congruence**
10) $ABCD$ is a rhombus.	10) ? **Definition of rhombus**

Name ______________________ Class ____________ Date ____________

6-6 Additional Vocabulary Support

Trapezoids and Kites

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
trapezoid	A *trapezoid* is a quadrilateral with one pair of parallel sides.	
legs of a trapezoid	**1. The *legs of a trapezoid* are the non-parallel sides.**	$\overline{TP}$ and $\overline{RA}$ 
bases of a trapezoid	**2. The *bases of a trapezoid* are the parallel sides.**	$\overline{TR}$ and $\overline{PA}$ 
isosceles trapezoid	An *isosceles trapezoid* is a trapezoid with legs that are congruent.	**3.** 
base angles	**4. The *base angles* are the angles that share the base of a trapezoid.**	$\angle A$ and $\angle B$ or $\angle C$ and $\angle D$ 
kite	A *kite* is a quadrilateral with two pairs of consecutive, congruent sides. In a kite, no opposite sides are congruent.	**5.** 
midsegment of a trapezoid	**6. The *midsegment of a trapezoid* is the segment that joins the midpoints of the legs.**	

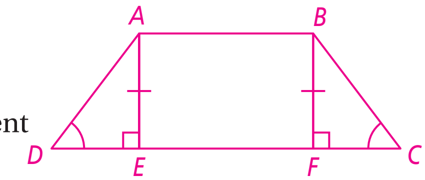
Name ______________________ Class ____________ Date ____________

6-6 Think About a Plan

Trapezoids and Kites

Prove the converse of Theorem 6-19: If a trapezoid has a pair of congruent base angles, then the trapezoid is isosceles.

Understanding the Problem



1. To help you solve this problem, draw trapezoid *ABCD* in the space to the right. Mark the base angles congruent and label the vertices so that $\overline{AB} \parallel \overline{CD}$.

2. Which two sides of the trapezoid do you need to prove congruent? **$\overline{AD}$ and $\overline{BC}$**

Planning the Solution

3. What type of polygons can you construct inside the trapezoid to prove that $\overline{AD} \cong \overline{BC}$? **a rectangle and right triangles**

4. What makes a set of lines parallel? Describe the relationship between two segments that are perpendicular to a pair of parallel lines and have endpoints on the lines.

   Parallel lines are always the same distance apart. The segments must be the same length.

5. In your diagram draw $\overline{AE} \perp \overline{DC}$ and $\overline{BF} \perp \overline{DC}$. What must be true about the length of these segments? Mark the diagram appropriately.

   They must be congruent because they are perpendicular to the same set of parallel lines and have endpoints on the lines.

6. What must be true of $\angle AED$ and $\angle BFC$? Explain your answer, and then mark the diagram appropriately.

   They are congruent because they are both 90° angles.

Getting an Answer

7. How can you prove that $\triangle AED \cong \triangle BFC$? Explain.

   AAS Theorem: It is given that $\angle D \cong \angle C$; $\angle AED \cong \angle BFC$ because they are both right angles, and $\overline{AE} \cong \overline{BF}$ because both are perpendicular to the same set of parallel lines with endpoints on the lines.

8. How does this allow you to prove that $\overline{AD} \cong \overline{BC}$? **CPCTC**

Name ______________________ Class ____________ Date ____________

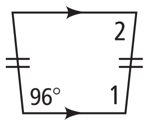
6-6 Practice

Form G

Trapezoids and Kites

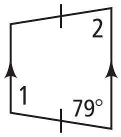
Find the measures of the numbered angles in each isosceles trapezoid.

1.



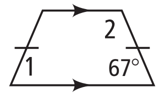
96; 84

2.



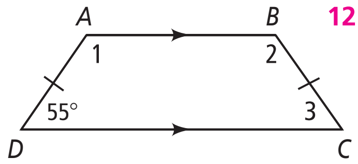
101; 79

3.



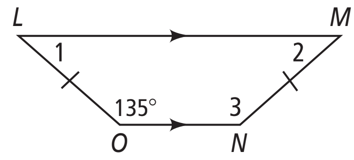
67; 113

4.



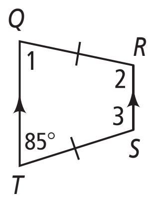
125; 125; 55

5.



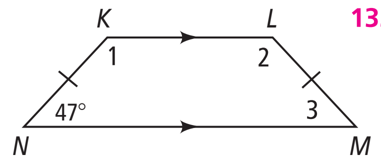
45; 45; 135

6.



85; 95; 95

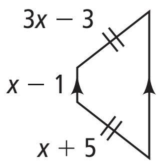
7.



133; 133; 47

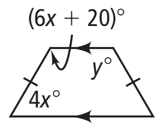
Algebra **Find the value(s) of the variable(s) in each isosceles trapezoid.**

8.



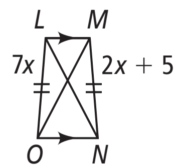
4

9.



16; 116

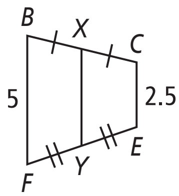
10.



1

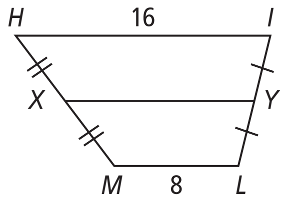
Find *XY* in each trapezoid.

11.



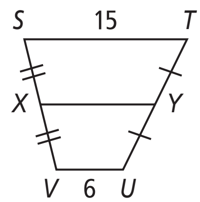
3.75

12.



12

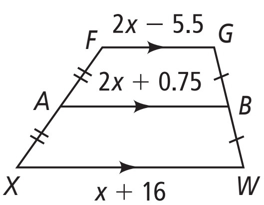
13.



10.5

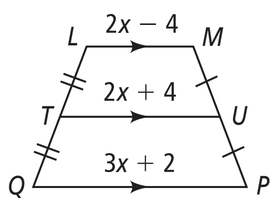
Algebra **Find the lengths of the segments with variable expressions.**

14.



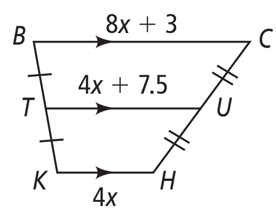
12.5; 18.75; 25

15.



16; 24; 32

16.



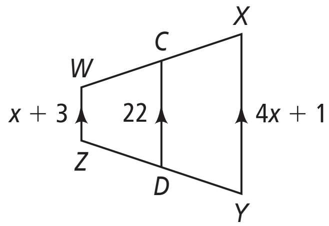
12; 19.5; 27

6-6 Practice (continued)

Form G

Trapezoids and Kites

17. $\overline{CD}$ is the midsegment of trapezoid *WXYZ*.



a. What is the value of *x*? **8**

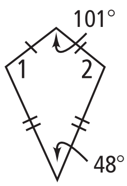
b. What is *XY*? **33**

c. What is *WZ*? **11**

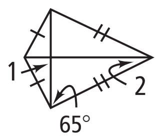
18. **Reasoning** The diagonals of a quadrilateral form two acute and two obtuse angles at their intersection. Is this quadrilateral a kite? Explain. **No; the diagonals of a kite are perpendicular.**

19. **Reasoning** The diagonals of a quadrilateral form right angles and its side lengths are 4, 4, 6, and 6. Could this quadrilateral be a kite? Explain.
Yes; if the adjacent sides are congruent, this would be a kite.

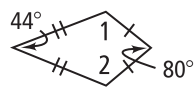
Find the measures of the numbered angles in each kite.

20. 

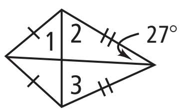
105.5; 105.5

21. 

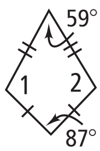
90; 25

22. 

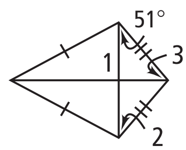
118; 118

23. 

90; 63; 63

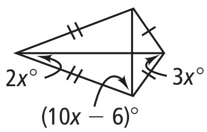
24. 

107; 107

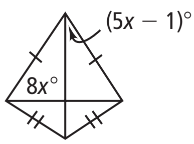
25. 

90; 51; 39

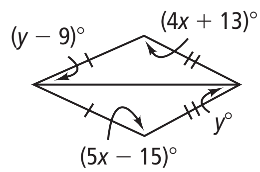
Algebra **Find the value(s) of the variable(s) in each kite.**

26. 

8

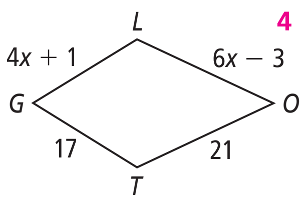
27. 

7

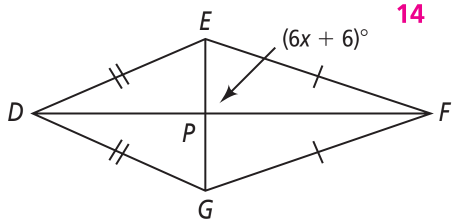
28. 

28; 32

For which value of *x* is each figure a kite?

29. 

4

30. 

14

Name ______________________________ Class ____________ Date ____________

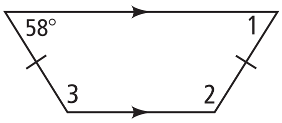
6-6 Practice

Form K

Trapezoids and Kites

Find the measures of the numbered angles in each isosceles trapezoid.

1.



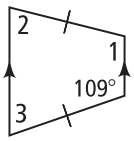
58; 122; 122

To start, identify which angles are congruent to and supplementary to the known angle.

∠ [1] is congruent to the 58° angle.

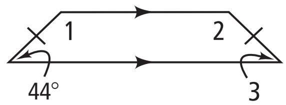
∠ [2] and ∠ [3] are supplementary to the 58° angle.

2.



109; 71; 71

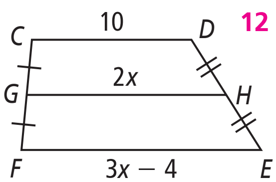
3.



136; 136; 44

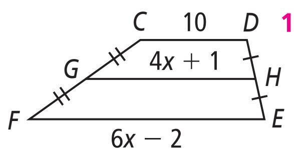
Find *GH* in each trapezoid.

4.



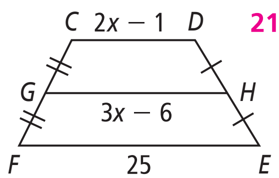
12

5.



13

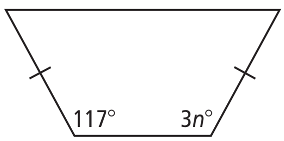
6.



21

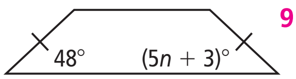
Algebra **Find the value of the variable in each isosceles trapezoid.**

7.



39

8.

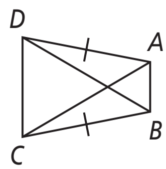


9

9. $AC = x + 5$

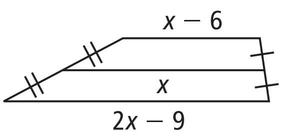
$BD = 2x - 2$

7



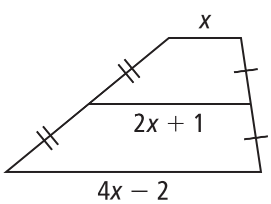
Algebra **Find the lengths of the segments with variable expressions.**

10.



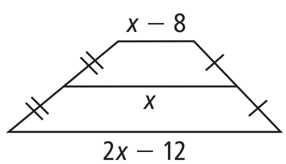
9; 15; 21

11.



4; 9; 14

12.



12; 20; 28

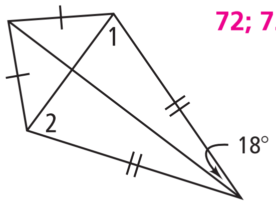
13. Open-Ended The midsegment of a trapezoid is 16 in. List three possible pairs of lengths for the bases of the trapezoid.

Answers may vary. Accept any pair of lengths that have a sum of 32.

6-6 Practice (continued) *Form K*

Trapezoids and Kites

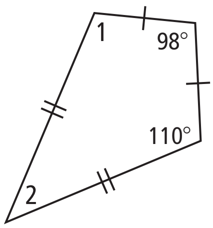
Find the measures of the numbered angles in each kite.

14. 

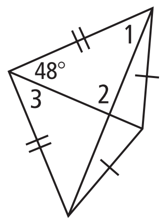
72; 72

To start, since the diagonals of a kite are perpendicular and the angle measures of a triangle add up to 180, write an equation with $m\angle 1$.

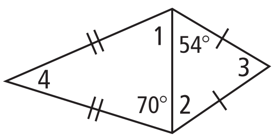
$m\angle 1 + \boxed{90} + \boxed{18} = 180$

15. 

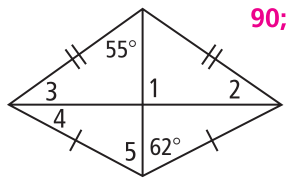
110; 42

16. 

42; 90; 48

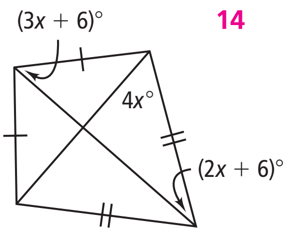
17. 

70; 54; 72; 40

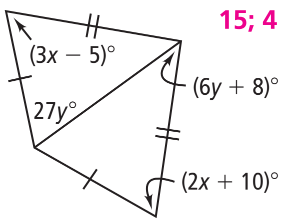
18. 

90; 35; 35; 28; 62

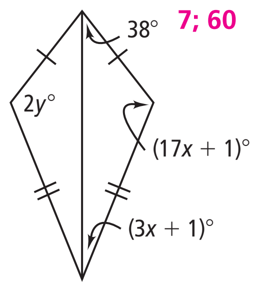
Algebra **Find the value(s) of the variable(s) in each kite.**

19. 

14

20. 

15; 4

21. 

7; 60

Determine whether each statement is *true* or *false*. Justify your response.

22. All kites are quadrilaterals. True; all kites have four sides.

23. A kite is a parallelogram. False; the opposite sides of a kite are not ∥.

24. A kite can have congruent diagonals. True; a kite may have congruent diagonals as long as only one diagonal is bisected.

25. Both diagonals of a kite bisect angles at the vertices.
False; one diagonal bisects angles at the vertices but the other diagonal does not.

Name ______________________ Class ____________ Date ____________

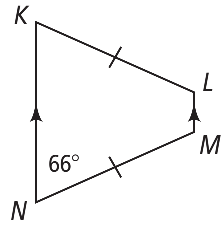
6-6 Standardized Test Prep

Trapezoids and Kites

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. In the isosceles trapezoid at the right, what is the measure of $\angle L$? **C**



Ⓐ 24 Ⓒ 114

Ⓑ 66 Ⓓ 132

2. What is true about the diagonals in an isosceles trapezoid? **F**

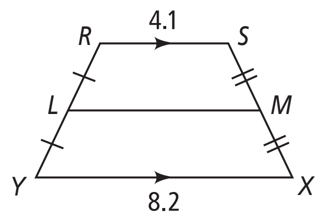
Ⓕ They are congruent.

Ⓖ They are perpendicular.

Ⓗ They are congruent and perpendicular.

Ⓘ The length of each diagonal is equal to half the sum of the bases.

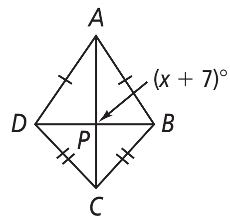
3. $\overline{LM}$ is the midsegment of trapezoid *RSXY*. What is *LM*? **C**



Ⓐ 4.1 Ⓒ 6.15

Ⓑ 6 Ⓓ 12.3

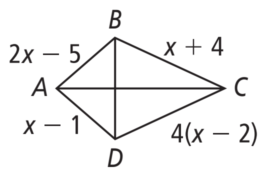
4. For which value of *x* is *ABCD* a kite? **I**



Ⓕ 23 Ⓗ 73

Ⓖ 33 Ⓘ 83

5. **Algebra** What is the value of *x* in kite *ABCD* at the right? **B**



Ⓐ 2 Ⓒ 8

Ⓑ 4 Ⓓ 16

Short Response

6. A quadrilateral has diagonals that are congruent and bisect opposite pairs of angles. Could this quadrilateral be a kite? Explain. **[2] No; it is a square, and in squares there are four congruent sides rather than two pairs of adjacent congruent sides. [1] correct answer with no explanation [0] no answer or wrong answer based on an inappropriate plan**

Name ______________________ Class ____________ Date ____________

6-6 Enrichment

Trapezoids and Kites

Trapezoid and Kite Angle Measures

The diagram below shows a kite (*WXYZ*) inscribed within an isosceles trapezoid (*ABCD*). $\overline{WY}$ is the midsegment of the trapezoid.

$AD = BC$

$\overline{XZ} \perp \overline{AB}$

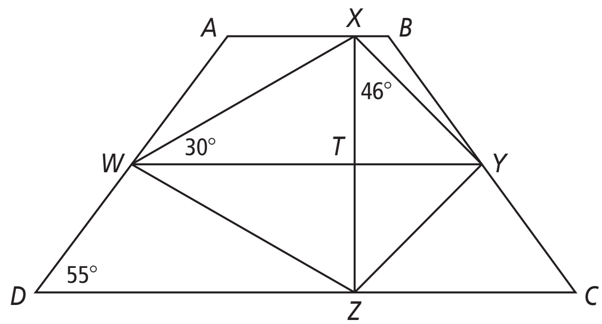
$m\angle WDZ = 55$

$m\angle XWT = 30$

$m\angle TXY = 46$

$WY = 12.6$

$BY = 4.24$



Use the properties of kites, isosceles trapezoids, triangles, and parallel lines to find the measures of the following angles.

1. $\angle XTY$ 90
2. $\angle XYT$ 44
3. $\angle BXY$ 44
4. $\angle XBY$ 125
5. $\angle YCZ$ 55
6. $\angle YTZ$ 90
7. $\angle TZY$ 46
8. $\angle WXT$ 60
9. $\angle TYZ$ 44
10. $\angle TWZ$ 30
11. $\angle WZT$ 60
12. $\angle DZW$ 30
13. $\angle DWZ$ 95
14. $\angle WAX$ 125
15. $\angle AWX$ 25
16. $\angle AXW$ 30
17. $\angle XYB$ 11
18. $\angle XTW$ 90
19. $\angle WZY$ 106
20. $\angle ZTW$ 90
21. $\angle CYZ$ 81
22. $\angle CZY$ 44
23. What is $AB + DC$? 25.2
24. What is WD? 4.24
25. What is the perimeter of the trapezoid? 42.16

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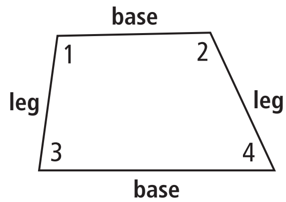
6-6 Reteaching

Trapezoids and Kites

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. The two parallel sides are called *bases.* The two nonparallel sides are called *legs.*

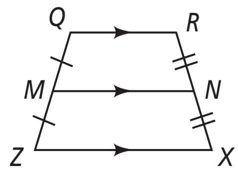


A pair of base angles share a common base.

$\angle 1$ and $\angle 2$ are one pair of base angles.

$\angle 3$ and $\angle 4$ are a second pair of base angles.

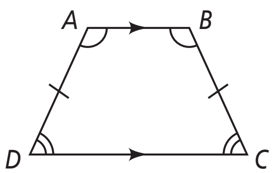
In any trapezoid, the *midsegment* is parallel to the bases. The length of the midsegment is half the sum of the lengths of the bases.



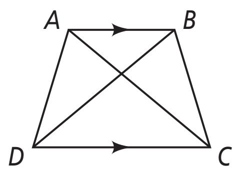
$$MN = \frac{1}{2}(QR + ZX)$$

An *isosceles trapezoid* is a trapezoid in which the legs are congruent. An isosceles trapezoid has some special properties:

Each pair of base angles is congruent.



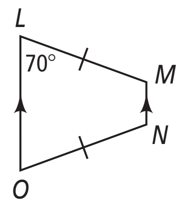
The diagonals are congruent.



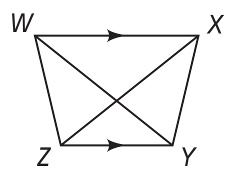
$AC = BD$

Exercises

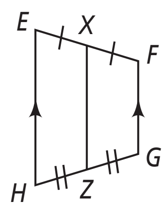
1. In trapezoid $LMNO$, what is the measure of $\angle OLM$? **70**
What is the measure of $\angle LMN$? **110**



2. $WXYZ$ is an isosceles trapezoid and $WY = 12$. What is XZ? **12**



3. $\overline{XZ}$ is the midsegment of trapezoid $EFGH$. If $FG = 8$ and $EH = 12$, what is XZ? **10**



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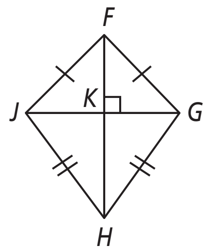
6-6 Reteaching (continued)

Trapezoids and Kites

A *kite* is a quadrilateral in which two pairs of consecutive sides are congruent and no opposite sides are congruent.

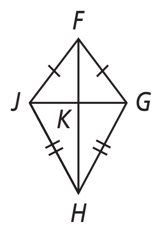
In a kite, the diagonals are perpendicular. The diagonals look like the crossbars in the frame of a typical kite that you fly.

Notice that the sides of a kite are the hypotenuses of four right triangles whose legs are formed by the diagonals.



Problem

Write a two-column proof to identify three pairs of congruent triangles in kite *FGHJ*.



Statements	Reasons
1) $m\angle FKG = m\angle GKH = m\angle HKJ = m\angle JKF = 90$	1) Theorem 6-22
2) $\overline{FG} \cong \overline{FJ}$	2) Given
3) $\overline{FK} \cong \overline{FK}$	3) Reflexive Property of Congruence
4) $\triangle FKG \cong \triangle FKJ$	4) HL Theorem
5) $\overline{JK} \cong \overline{KG}$	5) CPCTC
6) $\overline{KH} \cong \overline{KH}$	6) Reflexive Property of Congruence
7) $\triangle JKH \cong \triangle GKH$	7) SAS Postulate
8) $\overline{JH} \cong \overline{GH}$	8) Given
9) $\overline{FH} \cong \overline{FH}$	9) Reflexive Property of Congruence
10) $\triangle FJH \cong \triangle FGH$	10) SSS Postulate

So $\triangle FKG \cong \triangle FKJ$, $\triangle JKH \cong \triangle GKH$, and $\triangle FJH \cong \triangle FGH$.

Exercises

In kite *FGHJ* in the problem, $m\angle JFK = 38$ and $m\angle KGH = 63$. Find the following angle and side measures.

4. $m\angle FKJ$ 90 **5.** $m\angle FJK$ 52 **6.** $m\angle FKG$ 90

7. $m\angle KFG$ 38 **8.** $m\angle FGK$ 52 **9.** $m\angle GKH$ 90

10. $m\angle KHG$ 27 **11.** $m\angle KJH$ 63 **12.** $m\angle JHK$ 27

13. If $FG = 4.25$, what is JF? 4.25

14. If $HG = 5$, what is JH? 5

15. If $JK = 8.5$, what is GJ? 17

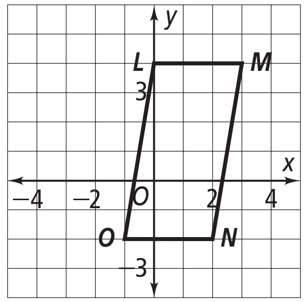
Name ______________________ Class ____________ Date ____________

6-7 Additional Vocabulary Support

Polygons in the Coordinate Plane

Problem

Is quadrilateral *LMNO* a parallelogram?



$\overline{LM}$ and $\overline{ON}$, $\overline{LO}$ and $\overline{MN}$ — Identify opposite sides.

Slope of $\overline{LM} = \left(\frac{4-4}{3-0}\right) = \frac{0}{3} = 0$ — Find the slope of $\overline{LM}$.

Slope of $\overline{ON} = \left(\frac{-2-(-2)}{2-(-1)}\right) = \frac{0}{3} = 0$ — Find the slope of $\overline{ON}$.

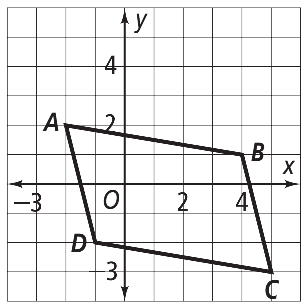
Slope of $\overline{LO} = \left(\frac{4-(-2)}{0-(-1)}\right) = \frac{6}{1} = 6$ — Find the slope of $\overline{LO}$.

Slope of $\overline{MN} = \left(\frac{4-(-2)}{3-2}\right) = \frac{6}{1} = 6$ — Find the slope of $\overline{MN}$.

$0 = 0, 6 = 6$ — Compare the slopes. Opposite sides have the same slope, so opposite sides are parallel. Figure *LMNO* is a parallelogram.

Exercise

Is quadrilateral *ABCD* a parallelogram?



$\overline{AD}$ and $\overline{BC}$, $\overline{AB}$ and $\overline{DC}$ **Identify opposite sides.**

Slope $\overline{AD} = \left(\frac{2-(-2)}{-2-(-1)}\right) = \frac{4}{-1} = -4$ **Find the slope of $\overline{AD}$.**

Slope $\overline{BC} = \left(\frac{1-(-3)}{4-5}\right) = \frac{4}{-1} = -4$ **Find the slope of $\overline{BC}$.**

Slope $\overline{AB} = \left(\frac{2-1}{-2-4}\right) = -\frac{1}{6}$ **Find the slope of $\overline{AB}$.**

Slope $\overline{DC} = \left(\frac{-2-(-3)}{-1-5}\right) = -\frac{1}{6}$ **Find the slope of $\overline{DC}$.**

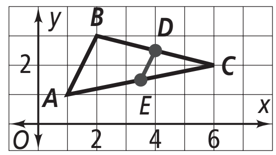
$-4 = -4, -\frac{1}{6} = -\frac{1}{6}$ **Compare the slopes. Opposite sides have the same slope, so opposite sides are parallel. Figure *ABCD* is a parallelogram.**

Name ______________________ Class ____________ Date ____________

6-7 Think About a Plan

Polygons in the Coordinate Plane

$\overline{DE}$ is a midsegment of $\triangle ABC$ at the right. Show that the Triangle Midsegment Theorem holds true for $\triangle ABC$.



Understanding the Problem

1. What does the Triangle Midsegment Theorem state?

If a segment joins the midpoints of two sides of a triangle, the segment is parallel to the third side and is one-half its length.

2. What do you need to prove to show that the Triangle Midsegment Theorem holds true for $\triangle ABC$? **You must prove that $\overline{DE} \parallel \overline{BA}$ and that $2DE = BA$.**

Planning the Solution

3. What is always true of the slope of parallel lines? **the slopes are equal**

4. Which formula can you use to find the length of a segment on the coordinate plane? **Distance Formula**

5. What are the coordinates for D and E? **$D(4, 2.5)$ and $E(3.5, 1.5)$**

Getting an Answer

6. Find the slope of segments $\overline{DE}$ and $\overline{BA}$ below.

Slope of $\overline{DE}$: $m = \dfrac{\boxed{2.5} - \boxed{1.5}}{\boxed{4} - \boxed{3.5}} = \dfrac{\boxed{1}}{\boxed{.5}} = \boxed{2}$

Slope of $\overline{BA}$: $m = \dfrac{\boxed{3} - \boxed{1}}{\boxed{2} - \boxed{1}} = \dfrac{\boxed{2}}{\boxed{1}} = \boxed{2}$

7. Are segments $\overline{DE}$ and $\overline{BA}$ parallel? **yes**

8. Find the length of segments $\overline{DE}$ and $\overline{BA}$ below.

Length of $\overline{DE}$: $\sqrt{(\square - \square)^2 + (\square - \square)^2} = \square$

$\sqrt{(4 - 3.5)^2 + (2.5 - 1.5)^2} = \sqrt{1.25}$

Length of $\overline{BA}$: $\sqrt{(\square - \square)^2 + (\square - \square)^2} = \square$

$\sqrt{(2 - 1)^2 + (3 - 1)^2} = \sqrt{5} = \sqrt{4 \times 1.25} = 2\sqrt{1.25}$

9. Does the Triangle Midsegment Theorem hold true for $\triangle ABC$? Explain.

Yes; $\overline{DE} \parallel \overline{BA}$ and the length of $\overline{DE}$ is one-half the length of $\overline{BA}$.

Name ______________________ Class ____________ Date ____________

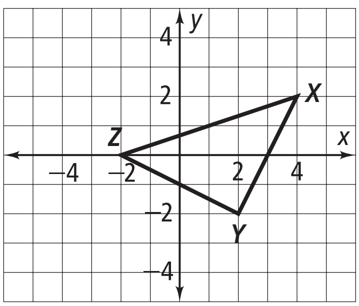
6-7 Practice

Form G

Polygons in the Coordinate Plane

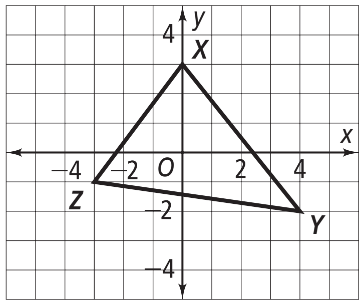
Determine whether △*XYZ* is *scalene, isosceles,* or *equilateral.*

1.



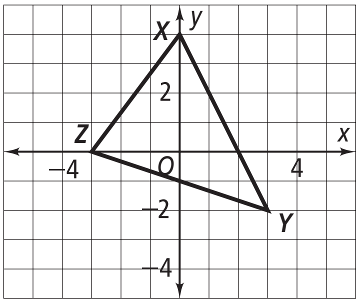
isosceles

2.



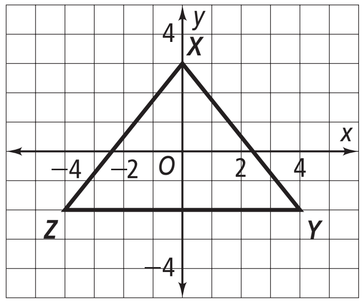
scalene

3.



scalene

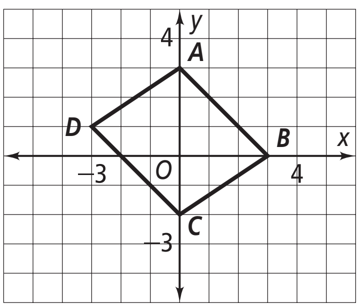
4.



isosceles

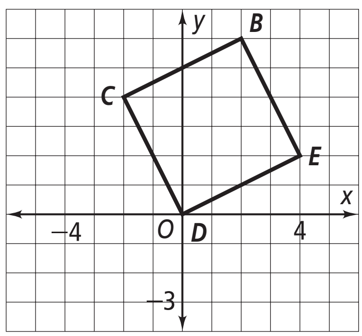
What is the most precise classification of the quadrilateral formed by connecting in order the midpoints of each figure below?

5.



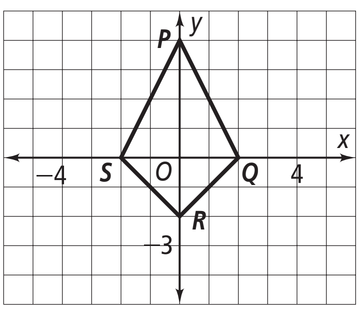
parallelogram

6.



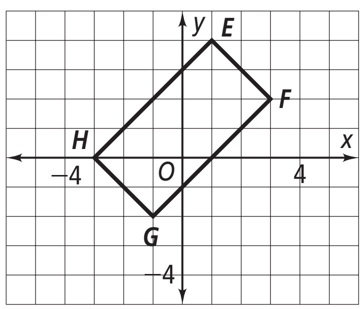
square

7.



rectangle

8.



rhombus

6-7 Practice (continued)

Form G

Polygons in the Coordinate Plane

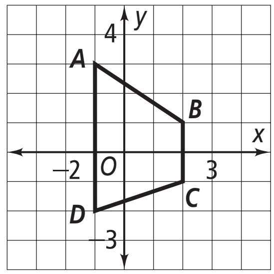
9. Writing Describe two ways in which you can show whether a parallelogram in the coordinate plane is a rectangle.

Determine whether the diagonals are congruent using the Distance Formula or determine if consecutive sides are perpendicular using the Slope Formula.

10. Writing Describe how you can show whether a quadrilateral in the coordinate plane is a kite.

Determine whether two pairs of consecutive sides are congruent, using the Distance Formulas, and that the pairs are not congruent to each other.

Use the trapezoid at the right for Exercises 11 and 12.



11. Is the trapezoid an isosceles trapezoid? Explain.

No; no sides are congruent.

12. Is the quadrilateral formed by connecting the midpoints of the trapezoid a parallelogram, rhombus, rectangle, or square? Explain.

Parallelogram; the slopes of the opposite sides are equal, but adjacent sides are not perpendicular and the sides are not all congruent.

Determine the most precise name for each quadrilateral. Then find its area.

13. $A(-6, 3)$, $B(-2, 0)$, $C(-2, -5)$, $D(-6, -2)$ **rhombus; 20 units^2**

14. $A(1, 8)$, $B(4, 6)$, $C(1, -2)$, $D(-2, 0)$ **parallelogram; 30 units^2**

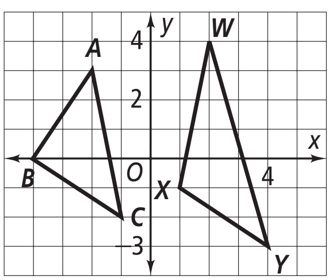
15. $A(3, 4)$, $B(8, 1)$, $C(2, -9)$, $D(-3, -6)$ **rectangle; 68 units^2**

16. $A(0, -1)$, $B(1, 4)$, $C(4, 3)$, $D(3, -2)$ **parallelogram; 16 units^2**

17. $A(-5, 14)$, $B(-2, 11)$, $C(-5, 8)$, $D(-8, 11)$ **square; 18 units^2**

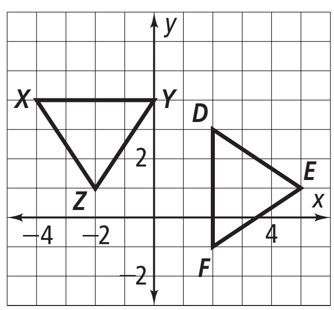
Determine whether the triangles are congruent. Explain.

18.



No; corresponding sides not ≅.

19.



Yes; corr. sides are ≅.

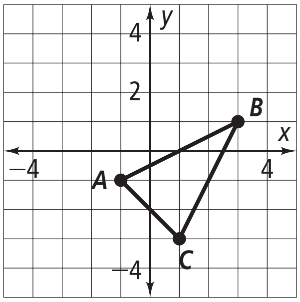
6-7 Practice

Form K

Polygons in the Coordinate Plane

Determine whether △*ABC* is *scalene, isosceles,* or *equilateral.* Explain.

1.

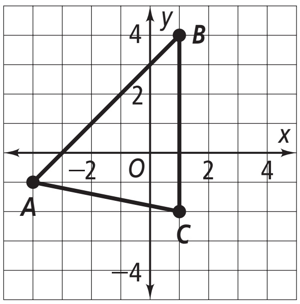


To start, determine the vertices of the triangle. Then use the Distance Formula to find the length of each side.

$A(-1, -1)$, $B(3, 1)$, $C(\boxed{1}, \boxed{-3})$

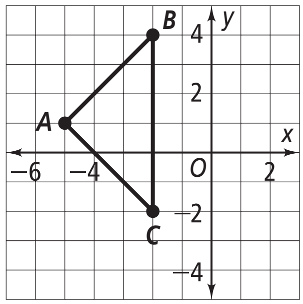
isosceles; *AB* = *CB*

2.



Scalene; no sides are congruent.

3.



isosceles; *AC* = *AB*

Determine whether the parallelogram is a *rhombus, rectangle, square,* or *none.* Explain.

4. $(-3, -1)$, $(-3, 2)$, $(1, 1)$, $(1, -2)$
none

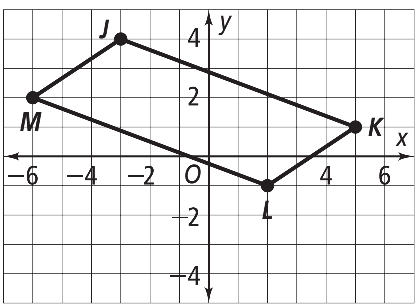
5. $(-5, 2)$, $(-3, 4)$, $(-3, 0)$, $(-1, 2)$
Square; diagonals are ≅ and ⊥.

6. $(-2, -1)$, $(-3, -3)$, $(1, -5)$, $(2, -3)$
Rectangle; sides are perpendicular, diagonals are ≅, opposite sides are ≅.

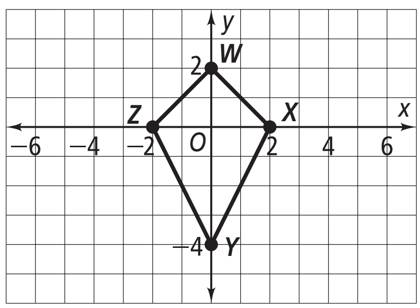
7. $(-6, -3)$, $(0, 5)$, $(10, 5)$, $(4, -3)$
Rhombus; diagonals are ⊥ and sides are ≅ but not ⊥.

What is the most precise classification of the quadrilateral formed by connecting in order the midpoints of each figure below?

8. ▱ *JKLM* parallelogram



9. kite *WXYZ* rectangle

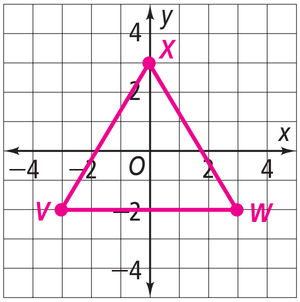


6-7 Practice (continued) — Form K

Polygons in the Coordinate Plane

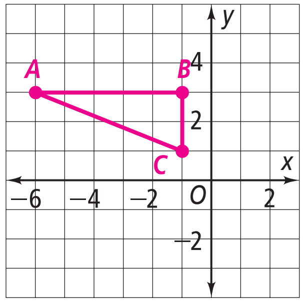
Graph and label each triangle with the given vertices. Determine whether each triangle is *scalene, isosceles,* or *equilateral.* Then tell whether each triangle is a right triangle.

10. $V(-3, -2)$, $W(3, -2)$, and $X(0, 3)$



isosceles; not right △

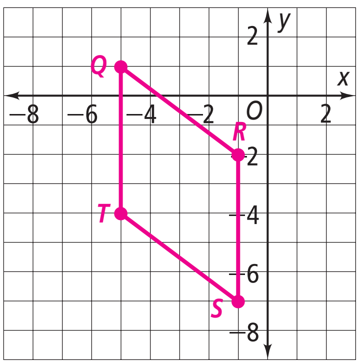
11. $A(-6, 3)$, $B(-1, 3)$, and $C(-1, 1)$



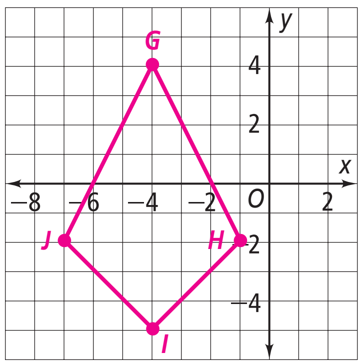
scalene; right △

Graph and label each quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral.

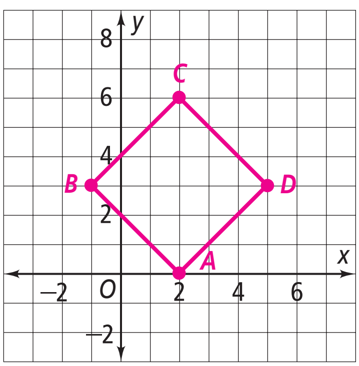
12. $Q(-5, 1)$, $R(-1, -2)$, $S(-1, -7)$, $T(-5, -4)$ rhombus



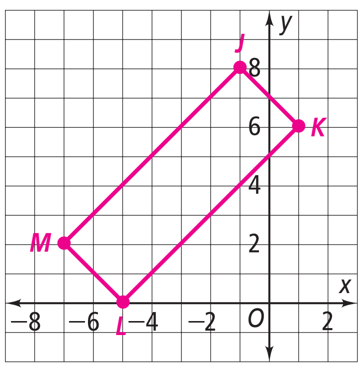
13. $G(-4, 4)$, $H(-1, -2)$, $I(-4, -5)$, $J(-7, -2)$ kite



14. $A(2, 0)$, $B(-1, 3)$, $C(2, 6)$, $D(5, 3)$ square



15. $J(-1, 8)$, $K(1, 6)$, $L(-5, 0)$, $M(-7, 2)$ rectangle



Name ______________________________ Class ____________ Date ____________

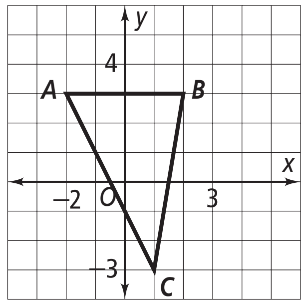
6-7 Standardized Test Prep

Polygons in the Coordinate Plane

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. What kind of triangle is this? D



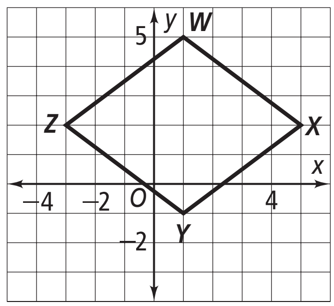
(A) right

(B) equilateral

(C) isosceles

(D) scalene

2. What is the most accurate description of the polygon at the right? F



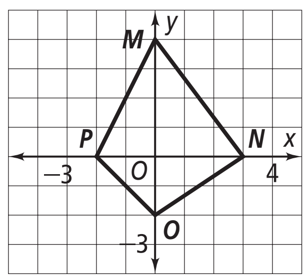
(F) rhombus

(G) square

(H) rectangle

(I) parallelogram

3. What is the most accurate description of the polygon at the right? D



(A) rhombus

(B) trapezoid

(C) kite

(D) quadrilateral

4. What kind of triangle is made by connecting the points $A(0, -6)$, $B(3, -6)$, and $C(3, -2)$? G

(F) equilateral

(G) right

(H) isosceles

(I) right and isosceles

Short Response

5. What type of quadrilateral is formed by connecting the points (0, 9), (3, 6), (0, 1), and (−3, 6)? Explain. **[2] Kite; two pairs of consecutive sides are congruent and no side is congruent to the side opposite it. [1] Kite; student explains that two pairs of consecutive sides are congruent or that no side is congruent to the side opposite it. [0] no answer or wrong answer based on an inappropriate plan**

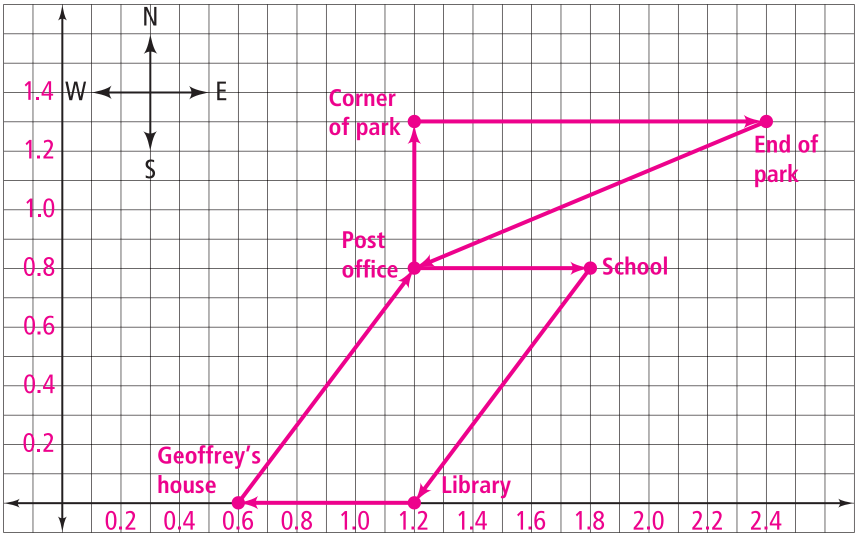
6-7 Enrichment

Polygons in the Coordinate Plane

Mapping a Course

The library is 0.6 mi due east from Geoffrey's house. The post office is 0.8 mi due north of the library. The park is north of the post office, and its southern boundary runs east-west. The school is due east of the post office. Geoffrey jogs along the same route each day through his town.

Geoffrey runs from his house directly to the post office. He turns left, and runs 0.5 mi north to the corner of the park. There he turns right, and runs 1.2 mi to the end of the park. From there he returns along a straight line southwest back to the post office. There he turns left and runs 0.6 mi due east to the school. At the school, he turns right and returns on a straight path southwest to the library. From the library, he runs straight home.



1. Use the coordinate grid above. Set every linear unit equal to 0.1 mi. For example, the distance between 0 and 2 on the *x*-axis is 0.2 mi. Plot out Geoffrey's running route. Mark each turn with a point and label, and use arrow marks to show Geoffrey's direction.

2. Geoffrey's path makes two polygons. What are these polygons? Give the most precise description of each. Explain. **Right scalene triangle and parallelogram; the triangle has a right angle and no ≅ sides; the parallelogram has parallel sides but no right angles and no adjacent sides ≅.**

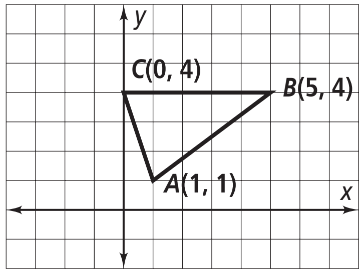
3. How far does Geoffrey run every day? Round your answer to the nearest 0.1 mi. **6.2 mi**

Name ______________________ Class ____________ Date ____________

6-7 Reteaching

Polygons in the Coordinate Plane

Below are some formulas that can help you classify figures on a coordinate plane.



To determine if line segments that form sides or diagonals are congruent, use the Distance Formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

In the figure at the right, the length of $\overline{AB}$ is $\sqrt{(5 - 1)^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = 5$.

In the figure above right the length of $\overline{BC}$ is $\sqrt{(5 - 0)^2 + (4 - 4)^2} = \sqrt{5^2 + 0^2} = 5$.

So, $\overline{AB} \cong \overline{BC}$. The figure is an isosceles triangle.

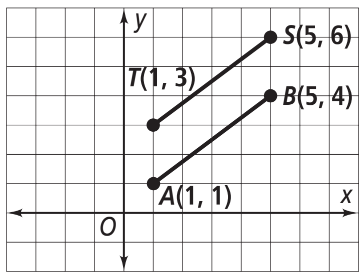
To find the midpoint of a side or diagonal, use the Midpoint Formula.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

In the figure above, the midpoint of $\overline{AB}$ is $\left(\frac{1 + 5}{2}, \frac{1 + 4}{2}\right) = \left(\frac{6}{2}, \frac{5}{2}\right) = (3, 2.5)$

To determine whether line segments that form sides or diagonals are parallel or perpendicular, use the Slope Formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$



In the figure at the right, the slope of $\overline{AB}$ is $\frac{(4 - 1)}{(5 - 1)} = \frac{3}{4}$.

The slope of $\overline{TS}$ is $\frac{(6 - 3)}{(5 - 1)} = \frac{3}{4}$. The line segments are parallel.

Lines with equal slopes are *parallel.*

Lines with slopes that have a product of -1 are *perpendicular.*

Exercises

1. How could you use the formulas to determine if a polygon on a coordinate plane is a rhombus? **Answers may vary. Sample: Use the Distance Formula to prove that the sides are all the same length.**
2. How could you use the formulas to determine if a trapezoid on a coordinate plane is isosceles? **Answers may vary. Sample: Use the Distance Formula to prove that the non-parallel pair of sides are congruent.**
3. How could you use the formulas to determine if a quadrilateral on a coordinate plane is a kite? **Answers may vary. Sample: Use the Distance Formula to show that there are two pairs of adjacent congruent sides and no side is congruent to the side opposite it.**

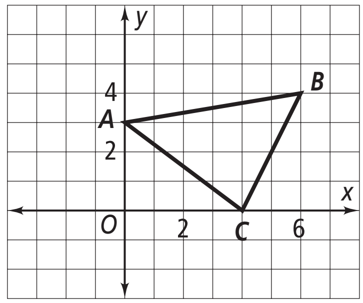
Name ________________ Class ________ Date ________

6-7 Reteaching (continued)

Polygons in the Coordinate Plane

Problem

Is $\triangle ABC$ scalene, isosceles, or equilateral?



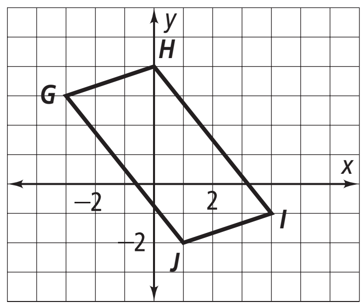
Find the lengths of the sides using the Distance Formula.

$$BA = \sqrt{(6)^2 + (1)^2} = \sqrt{36 + 1} = \sqrt{37}$$
$$BC = \sqrt{(2)^2 + (4)^2} = \sqrt{20}$$
$$CA = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

The sides are all different lengths. So, $\triangle ABC$ is scalene.

Problem

Is quadrilateral *GHIJ* a parallelogram?



Find the slopes of the opposite sides.

slope of $\overline{GH} = \frac{4 - 3}{0 - (-3)} = \frac{1}{3}$; slope of $\overline{JI} = \frac{-1 - (-2)}{4 - 1} = \frac{1}{3}$

slope of $\overline{HI} = \frac{-1 - 4}{4 - 0} = \frac{-5}{4}$; slope of $\overline{GJ} = \frac{-2 - 3}{1 - (-3)} = \frac{-5}{4}$

So, $\overline{JI} \parallel \overline{GH}$ and $\overline{HI} \parallel \overline{GJ}$. Therefore, *GHIJ* is a parallelogram.

Exercises

$\triangle JKL$ has vertices at $J(-2, 4)$, $K(1, 6)$, and $L(4, 4)$.

4. Determine whether $\triangle JKL$ is *scalene, isosceles,* or *equilateral.* Explain.
Isosceles; use the Distance Formula to find that $JK = KL = \sqrt{13}$, and $JL = 6$.

5. Determine whether $\triangle JKL$ is a right triangle. Explain.
No; use the Slope Formula. The slope of $\overline{JK} = \frac{6 - 4}{1 - (-2)} = \frac{2}{3}$, the slope of $\overline{KL} = \frac{6 - 4}{1 - 4} = -\frac{2}{3}$, and the slope of $\overline{JL} = \frac{4 - 4}{-2 - 4} = 0$, so no sides are perpendicular.

6. Trapezoid *ABCD* has vertices at $A(2, 1)$, $B(12, 1)$, $C(9, 4)$, and $D(5, 4)$. Which formula would help you find out if this trapezoid is isosceles? Is this an isosceles trapezoid? Explain.
Distance Formula; yes; the distances *AD* and *BC* are both $\sqrt{18}$ or $3\sqrt{2}$. Because these distances represent the legs, *ABCD* is isosceles.

Name ____________________ Class __________ Date __________

6-8 Additional Vocabulary Support

Applying Coordinate Geometry

For Exercises 1–4, match the word in Column A with its definition in Column B. The first one is done for you.

Column A	Column B
variable	a proof that uses coordinate geometry to show a theorem is true
1. position	numbers or variables that define the position of a point on the coordinate plane
2. coordinates	the place where two line segments meet
3. coordinate proof	a symbol or letter that represents an unknown number
4. vertex	the location of an object

For Exercises 5–7, match the term in Column A with the formula in Column B.

Column A	Column B
5. Distance Formula	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
6. Slope Formula	$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$
7. Midpoint Formula	$\frac{(y_2 - y_1)}{(x_2 - x_1)}$

For Exercises 8–10, match the formula in Column A with the problem in Column B that requires the formula.

Column A	Column B
8. Distance Formula	Find point *E* halfway between *A* and *B* on side $\overline{AB}$ of quadrilateral *ABCD*.
9. Slope Formula	Determine whether sides $\overline{AB}$ and $\overline{CD}$ on quadrilateral *ABCD* are parallel.
10. Midpoint Formula	Find *AB* for quadrilateral *ABCD* on the coordinate plane.

Name ______________________ Class ____________ Date ____________

6-8 Think About a Plan

Applying Coordinate Geometry

Plan the coordinate proof of the statement.

The diagonals of a rectangle bisect each other.

Know

1. What does it mean when we say that the diagonals bisect each other?

   They divide each other in half.

2. If the diagonals do bisect each other, what should the point of intersection be for each diagonal?

   the midpoint of that diagonal

Need

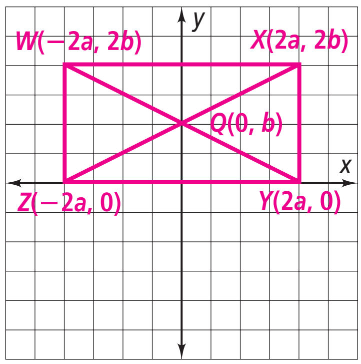
3. Which formula will you need to use to prove that the diagonals bisect each other?

   Midpoint Formula

4. Based on the formula you need to use, what type of number should you use for the coordinates of the vertices?

   multiples of 2

5. Draw rectangle *WXYZ* on the coordinate plane at the right. Include diagonals that intersect at point *Q*. Use variables to write the coordinates for each point.



6. What information is given?

   ***WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$.**

7. What must you prove?

   ***Q* is the midpoint of both $\overline{WY}$ and $\overline{XZ}$.**

Plan

8. How will you go about proving this?

   Use the Midpoint Formula to find the midpoints of $\overline{WY}$ and $\overline{XZ}$. If the coordinates of the midpoint of both lines are the coordinates of Point *Q*, the statement is proven.

Name ______________________ Class ____________ Date ____________

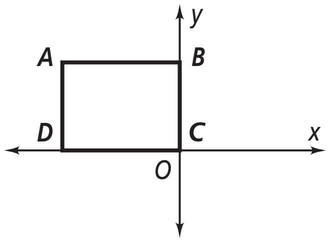
6-8 Practice

Form G

Applying Coordinate Geometry

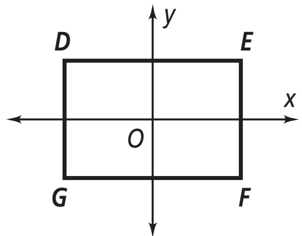
Algebra What are the coordinates of the vertices of each figure?

1. rectangle with base b and height h



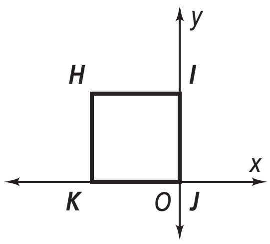
$A(-b, h)$; $B(0, h)$; $C(0, 0)$; $D(-b, 0)$

2. rectangle centered at the origin with base $2b$ and height $2h$



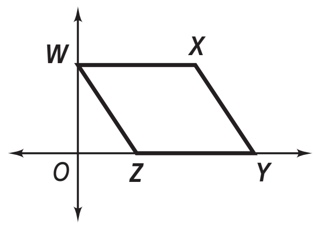
$E(b, h)$; $F(b, -h)$; $G(-b, -h)$; $D(-b, h)$

3. square with height x



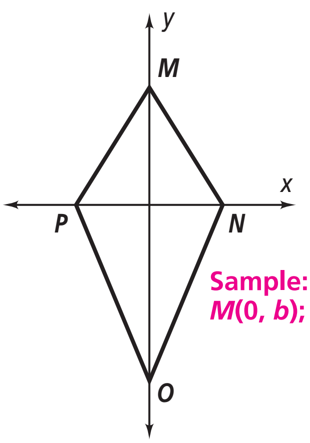
$H(-x, x)$; $I(0, x)$; $J(0, 0)$; $K(-x, 0)$

4. parallelogram with height m and point Z distance j from the origin



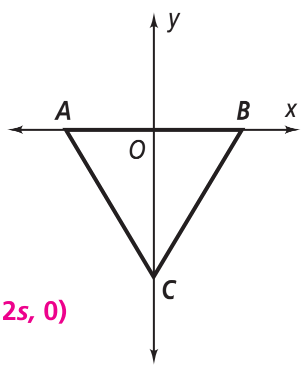
Sample: $W(0, m)$; $X(x - j, m)$; $Y(x, 0)$; $Z(j, 0)$

5. kite $MNOP$ where $PN = 4s$ and the y-axis bisects $\overline{PN}$



Sample: $M(0, b)$; $N(2s, 0)$; $O(0, -c)$; $P(-2s, 0)$

6. isosceles $\triangle ABC$ where $AB = 2n$ and the y-axis is the median



Sample: $A(-n, 0)$; $B(n, 0)$; $C(0, -y)$

7. How can you determine if a triangle on a coordinate grid is an isosceles triangle? **Use the Distance Formula to compare side lengths.**

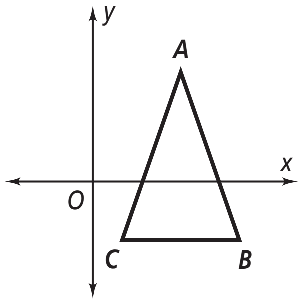
8. How can you determine if a parallelogram on a coordinate grid is a rhombus? **Answers may vary. Sample: Use the Slope Formula to determine if the diagonals are perpendicular.**

9. How can you determine if a parallelogram on a coordinate grid is a rectangle? **Answers may vary. Sample: Use the Distance Formula to determine if the diagonals are congruent.**

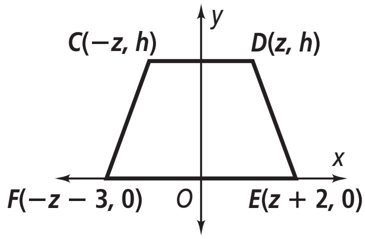
6-8 Practice (continued) *Form G*

Applying Coordinate Geometry

10. In the triangle at the right, A is at $(m + r, s)$, B is at $(2m, -p)$, and C is at $(2r, -p)$. Is this an isosceles triangle? Explain.
Yes; $AC = AB$. Both are equal to $\sqrt{m^2 - 2rm + r^2 + s^2 + 2sp + p^2}$.

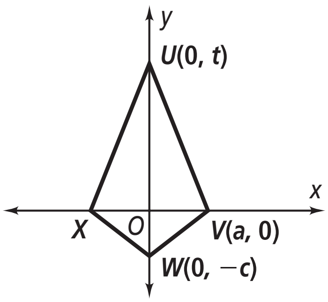


11. Is the trapezoid shown at the right an isosceles trapezoid? Explain.
No; the diagonals are not congruent. $CE = \sqrt{(2z + 2)^2 + h^2}$, $DF = \sqrt{(2z + 3)^2 + h^2}$

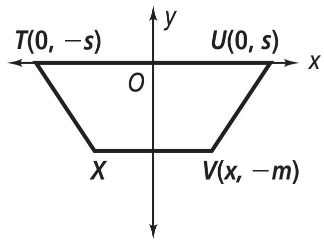


For Exercises 12 and 13, give the coordinates for point X without using any new variables.

12. Kite **$(-a, 0)$**



13. $TX = UV$ **$(-x, -m)$**

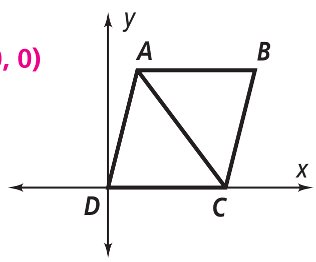


14. Plan a coordinate proof to show that either diagonal of a parallelogram divides the parallelogram into two congruent triangles.

a. Name the coordinates of parallelogram $ABCD$ at the right.
Answers may vary. Sample: $A(p, r)$; $B(p + m, r)$; $C(m, 0)$; $D(0, 0)$

b. What do you need to do to show that $\triangle ACD$ and $\triangle CAB$ are congruent?
Show that corresponding sides are ≅.

c. How will you determine that those parts are congruent?
Use the Distance Formula.



Classify each quadrilateral as precisely as possible.

15. $A(-3a, 3a)$, $B(3a, 3a)$, $C(3a, -3a)$, $D(-3a, -3a)$ **square**

16. $A(c, d + e)$, $B(2c, d)$, $C(c, d - 2e)$, $D(0, d)$ **kite**

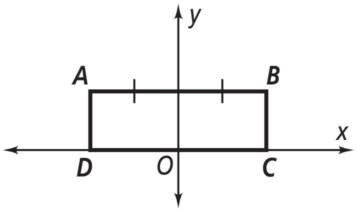
6-8 Practice

Form K

Applying Coordinate Geometry

Algebra What are the coordinates of the vertices of each figure?

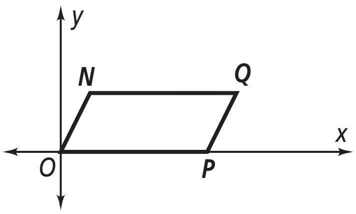
1. rectangle with base $2b$ and height h **$A(-b, h)$, $B(b, h)$, $C(b, 0)$, $D(-b, 0)$**



To start, identify the coordinates of C.

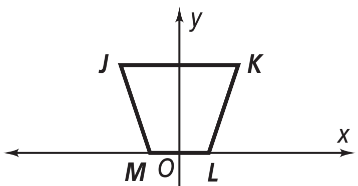
Because $CD = 2b$, the x-coordinate of C is $\frac{2b}{2}$, or **b**. C is on the x-axis, so its y-coordinate is **0**.

2. parallelogram with height a, and point P distance b from the origin



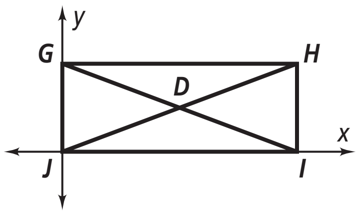
$N(c, a)$, $O(0, 0)$, $P(b, 0)$, $Q(b + c, a)$

3. isosceles trapezoid with base centered at the origin, with bases $4a$ and $2b$, and height c

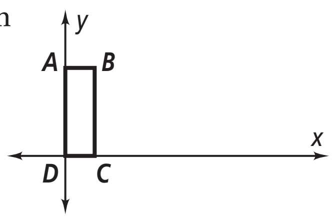


$J(-2a, c)$, $K(2a, c)$, $L(b, 0)$, $M(-b, 0)$

4. The diagram shows a rectangle with base a and height b. What are the coordinates of D, the point of intersection of the diagonals of rectangle $GHIJ$? Explain.
$(\frac{a}{2}, \frac{b}{2})$; these are the x-coordinate of the midpoint of GH and the y-coordinate of the midpoint of HI.



5. Plan a coordinate proof to show that a parallelogram is a rectangle.



a. Name the coordinates of ▱ $ABCD$ at the right with base b and height h.
$A(0, h)$, $B(b, h)$, $C(b, 0)$, $D(0, 0)$

b. Write the *Given* and *Prove* Statements.
Given: ▱ $ABCD$; Prove: $ABCD$ is a rectangle.

c. What information do you need to prove that the parallelogram is a rectangle?
Answers may vary. Sample: You need to prove that the adjacent sides are perpendicular. To do this, you need to show that the slopes of the lines of the adjacent sides are negative reciprocals.

6. Open-Ended Place a kite in the coordinate plane.
Check students' work. Kite could be placed so that the diagonals are the x- and y-axes.

Name ________________________ Class ____________ Date ____________

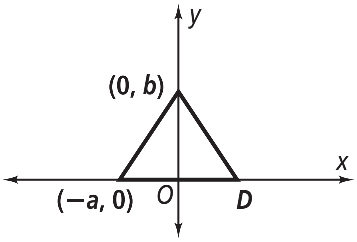
6-8 Practice (continued) — Form K

Applying Coordinate Geometry

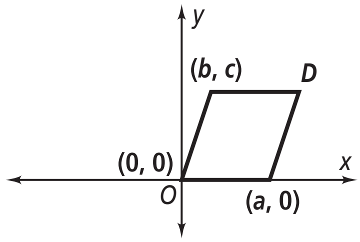
7. Reasoning A rhombus $QRST$ is centered at the origin with $QS = 4r$ and $RT = 4t$. What are the coordinates of each vertex?
$Q(-2r, 0)$, $R(0, 2t)$, $S(2r, 0)$, $T(0, -2t)$ or $Q(0, -2r)$, $R(2t, 0)$, $S(0, 2r)$, $T(-2t, 0)$

Give the coordinates for point D without using any new variables.

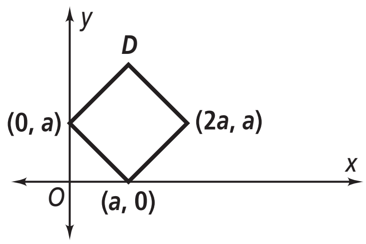
8. isosceles triangle **$(a, 0)$**



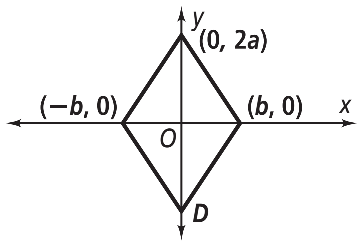
9. parallelogram **$(b + a, c)$**



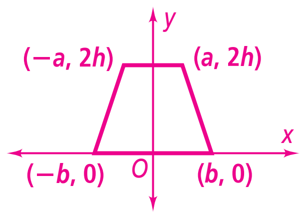
10. square **$(a, 2a)$**



11. rhombus **$(0, -2a)$**



12. a. Draw an isosceles trapezoid with height $2h$ and bases $2a$ and $2b$ so that the origin is the midpoint of the base with length $2b$.



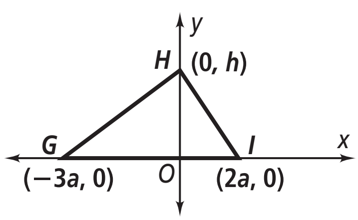
b. Give the coordinates of the vertices of the trapezoid.

c. Compute the length of the legs of the trapezoid.
$\sqrt{(a - b)^2 + (2h)^2}$

d. Find the slopes of the two legs of the trapezoid.
$\frac{2h}{a - b}$ and $\frac{2h}{-a + b}$

e. Find the midpoints of the legs of the trapezoid.
$\left(\frac{a + b}{2}, h\right)$; $\left(\frac{-a + (-b)}{2}, h\right)$

13. Reasoning Is the triangle at the right an isosceles triangle with base $\overline{GI}$? How can you tell without using the Distance Formula?
No; answers may vary. Sample: the points G and I are different distances from the origin.



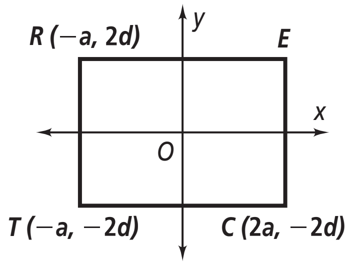
Name ______________________ Class ____________ Date ____________

6-8 Standardized Test Prep

Applying Coordinate Geometry

Multiple Choice

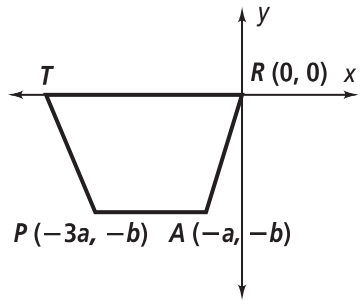
For Exercises 1–5, choose the correct letter.



1. Rectangle *RECT* is shown at the right. What are the coordinates of point *E*? **D**

(A) $(2a, d)$ (C) $(-2a, d)$

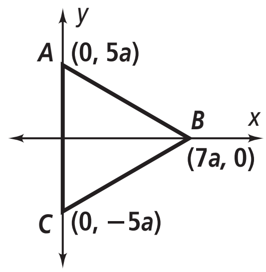
(B) $(a, 2d)$ (D) $(2a, 2d)$



2. Isosceles trapezoid *TRAP* is shown at the right. What are the coordinates of point *T*? **F**

(F) $(-4a, 0)$ (H) $(0, -4a)$

(G) $(-b, 0)$ (I) $(-3b, 0)$



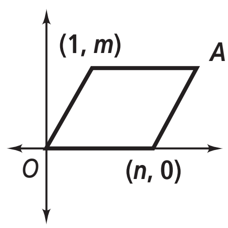
3. What type of triangle is shown at the right? **C**

(A) equilateral (C) isosceles

(B) right (D) scalene

4. What is the most precise description of a quadrilateral with coordinates $A(-a, b)$, $B(3a, b)$, $C(3a, -b)$, $D(-a, -b)$? **G**

(F) kite (G) rectangle (H) rhombus (I) square



5. Given the parallelogram at the right, what coordinates for point *A* can you write without using any new variables? **B**

(A) (n, m) (C) $(n + m, 1)$

(B) $(1 + n, m)$ (D) $(m, n + 1)$

Short Response

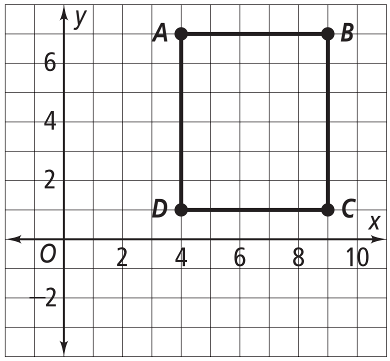
6. What type of quadrilateral is formed by connecting the points $(0, 0)$, $(3x, b)$, $(18x, b)$, and $(15x, 0)$? Explain.
[2] Sample: A parallelogram; opposite sides are ≅ and parallel, but diagonals are not ≅ and adjacent sides are not ≅. [1] Student proves it is a parallelogram, but not that it is not a special parallelogram. [0] no answer or wrong answer based on an inappropriate plan

6-8 Enrichment

Applying Coordinate Geometry

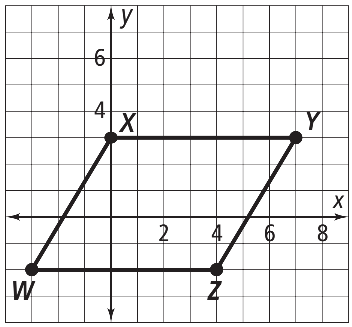
Coordinate Geometry, Introduction to Translations

Given rectangle *ABCD* at the right, slide the rectangle to the left so that point *A* is on the *y*-axis.



1. What are the new coordinates of *A*? **(0, 7)**
2. What are the new coordinates of *D*? **(0, 1)**
3. What are the new coordinates of *C*? **(5, 1)**
4. What are the new coordinates of *B*? **(5, 7)**
5. How have the coordinates of each point changed? Explain.
   The *x*-coordinate of each point has decreased by 4.

Given parallelogram *WXYZ* at the right, slide *WXYZ* so that point *X* is at the origin.



6. What are the new coordinates of *Y*? **(7, 0)**
7. What are the new coordinates of *W*? **(–3, –5)**
8. Find the midpoint of $\overline{WX}$ after the slide. **(–1.5, –2.5)**
9. How can you now find the coordinates of the midpoint before the slide?
   Add 3 to the *y*-coordinate.
10. What has happened to the coordinates of the parallelogram during the slide?
    The *y*-coordinate of each point has decreased by 3.
11. What has happened to the coordinates of the midpoint of $\overline{WX}$?
    The *y*-coordinate has decreased by 3.

Quadrilateral *ABCD* is on the coordinate grid, $A(-2a, 0)$, $B(-a, b)$, $C(a, b)$, and $D(2a, 0)$. Imagine that you slide *ABCD* *a* units to the right and *p* units up.

12. Classify the new quadrilateral as precisely as possible. **isosceles trapezoid**
13. What are the new coordinates of *A*? **$(-a, p)$**
14. What are the new coordinates of *B*? **$(0, b + p)$**
15. What are the new coordinates of *C*? **$(2a, b + p)$**
16. What are the new coordinates of *D*? **$(3a, p)$**
17. What has happened to the coordinates of the midpoint of $\overline{AB}$?
    The *x*-coordinate has increased by *a* and the *y*-coordinate has increased by *p*.
18. Does a slide change the slope of any of the line segments in the quadrilateral? **no**
19. Will any translation of a geometric figure change its classification or measurements? Explain. **No; a translation does not change the size or shape of a geometric figure, it only changes the location.**

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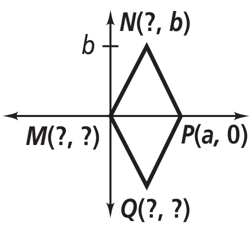
6-8 Reteaching

Applying Coordinate Geometry

You can use variables instead of integers to name the coordinates of a polygon in the coordinate plane.

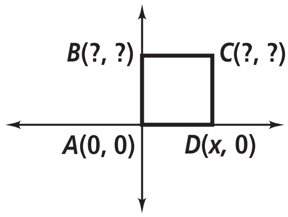
Problem

Use the properties of each figure to find the missing coordinates.



rhombus *MNPQ*

M is at the origin (0, 0). Because diagonals of a rhombus bisect each other, *N* has *x*-coordinate $\frac{a}{2}$. Because the *x*-axis is a horizontal line of symmetry for the rhombus, *Q* has coordinates $(\frac{a}{2}, -b)$.



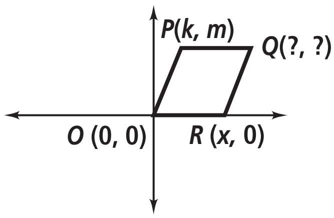
square *ABCD*

Because all sides are congruent, *D* has coordinate $(0, x)$. Because all angles are right, *C* has coordinates (x, x).

Exercises

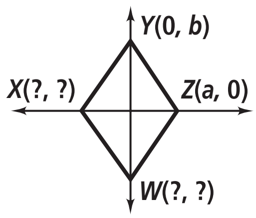
Use the properties of each figure to find the missing coordinates.

1. parallelogram *OPQR*



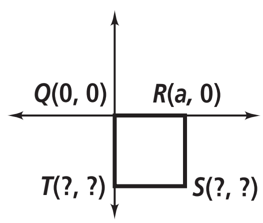
$Q(x + k, m)$

2. rhombus *XYZW*



$X(-a, 0)$; $W(0, -b)$

3. square *QRST*



$S(a, -a)$, $T(0, -a)$

4. A quadrilateral has vertices at $(a, 0)$, $(-a, 0)$, $(0, a)$, and $(0, -a)$. Show that it is a square.
Sample: Each side has a length of $a\sqrt{2}$, making the figure a rhombus. One pair of opposite sides has a slope of 1, and the other pair has a slope of –1. Because the product of the slopes is –1, the sides are perpendicular and the rhombus is a square.

5. A quadrilateral has vertices at $(a, 0)$, $(0, a + 1)$, $(-a, 0)$, and $(0, -a - 1)$. Show that it is a rhombus.
Each side has a length of $\sqrt{2a^2 + 2a + 1}$. Therefore, the figure is a rhombus.

6. Isosceles trapezoid *ABCD* has vertices $A(0, 0)$, $B(x, 0)$, and $D(k, m)$. Find the coordinates of *C* in terms of *x*, *k*, and *m*. Assume $\overline{AB} \parallel \overline{CD}$. **$C(x - k, m)$**

6-8 Reteaching (continued)

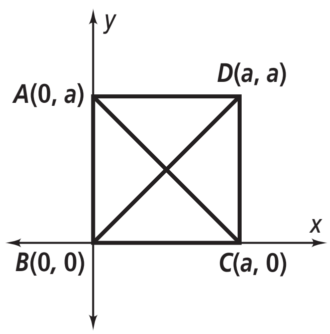
Applying Coordinate Geometry

You can use a *coordinate proof* to prove geometry theorems. You can use the Distance Formula, the Slope Formula, and the Midpoint Formula when writing coordinate proofs. With the Midpoint Formula, using multiples of two to name coordinates makes computation easier.

Problem

Plan a coordinate proof to show that the diagonals of a square are congruent.

Draw and label a square on a coordinate grid. In square $ABCD$, $AB = BC = CD = DA$. Draw in the diagonals, $\overline{AC}$ and $\overline{BD}$.



Prove that $AC = BD$. Use the Distance Formula.

$$CA = \sqrt{(0 - a)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2}$$

$$BD = \sqrt{(a - 0)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2}$$

So, $CA = BD$. The diagonals of the square are congruent.

Exercises

7. How would you use a coordinate proof to prove that the diagonals of a square are perpendicular?
Answers may vary. Sample: Use the Slope Formula to prove that the product of the slopes of the diagonals is −1.

8. How would you use a coordinate proof to prove that the diagonals of a rectangle are congruent?
Answers may vary. Sample: Use the Distance Formula to prove that the lengths of the diagonals are equal.

9. How would you use a coordinate proof to prove that if the midpoints of the sides of a trapezoid are connected they will form a parallelogram?
Answers may vary. Sample: Use the Midpoint Formula to find the midpoints, then use the Slope Formula to show that opposite sides in the new figure have equal slopes.

10. How would you use a coordinate proof to prove that the diagonals of a parallelogram bisect one another?
Answers may vary. Sample: Use the Midpoint Formula to show that the midpoints of the diagonals are the same point.

11. Classify quadrilateral $ABCD$ with vertices $A(0, 0)$, $B(a, -b)$, $C(c, -b)$, $D(a + c, 0)$ as precisely as possible. Explain.
isosceles trapezoid; one pair of sides parallel, other opposite pair of sides ≅, and diagonals ≅

12. Classify quadrilateral $FGHJ$ with vertices $F(a, 0)$, $G(a, 2c)$, $H(b, 2c)$, and $J(b, c)$ as precisely as possible. Explain.
Trapezoid; one pair of sides is parallel, the other pair of sides is not parallel.

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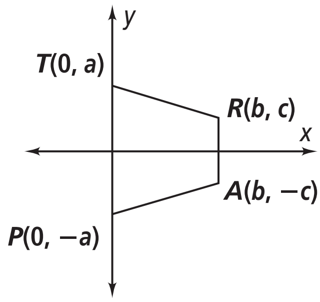
6-9 Additional Vocabulary Support

Proofs Using Coordinate Geometry

There are two sets of note cards that show that trapezoid *TRAP* is an isosceles trapezoid. The set on the left shows the statements and the set on the right shows the reasons. Write the statements and the reasons in the correct order.



Think Cards

- Show that TR = PA.
- Use the distance formula to find TR.
- Use the distance formula to find PA.

Write Cards

- $d = \sqrt{(c-a)^2 + (b-0)^2} = \sqrt{(c-a)^2 + b^2}$
- $d = \sqrt{(-a-(-c))^2 + (0-b)^2} = \sqrt{(c-a)^2 + b^2}$
- $\sqrt{(c-a)^2 + b^2} = \sqrt{(c-a)^2 + b^2}$

Think Answers may vary. Sample: **Write**

Think	Write
First, you should use the distance formula to find *TR*.	**Step 1** $d = \sqrt{(c-a)^2 + (b-0)^2} = \sqrt{(c-a)^2 + b^2}$
Then, you should use the distance formula to find *PA*.	**Step 2** $d = \sqrt{(-a-(-c))^2 + (0-b)^2} = \sqrt{(c-a)^2 + b^2}$
Last, you should show that *TR* = *PA*.	**Step 3** $\sqrt{(c-a)^2 + b^2} = \sqrt{(c-a)^2 + b^2}$

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6-9 Think About a Plan

Proofs Using Coordinate Geometry

Use coordinate geometry to prove the following statement.

The altitude to the base of an isosceles triangle bisects the base.

Understanding the Problem

1. What makes a triangle isosceles? **Two of its sides are congruent.**

2. What is an altitude?
the perpendicular segment that connects one vertex of a triangle to the opposite side

3. What does it mean when we say that the altitude bisects the base?
The altitude divides the base into two congruent halves.

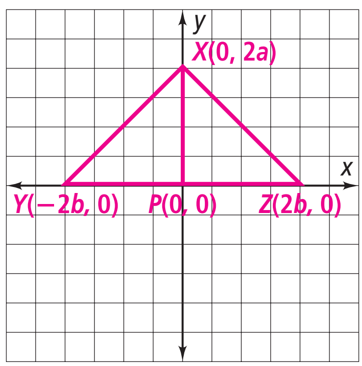
4. If the altitude bisects the base, what should the point of intersection between the altitude and the base represent? **the midpoint of the base**

Planning the Solution

5. Which formula will you need to use to prove that the altitude bisects the base?
Midpoint Formula

6. Based on the formula you chose, what type of numbers should the coordinates be? **multiples of 2**

7. Think about how you can draw the triangle on the coordinate plane so that the altitude will intersect $\overline{YZ}$ at (0, 0). Draw isosceles triangle XYZ such that $\overline{XY} \cong \overline{XZ}$. Draw altitude $\overline{XP}$. Use variables to write the coordinates for each point.



Getting an Answer

8. Use the Midpoint Formula to find the midpoint of $\overline{YZ}$.
$M = \left(\frac{? + ?}{2}, \frac{? + ?}{2}\right)$ $\left(\frac{-2b + 2b}{2}, \frac{0 + 0}{2}\right) = (0, 0)$

9. Does the altitude to the base of an isosceles triangle bisect the base? Explain.
Yes; the coordinates of the midpoint of the base are the same as the coordinates of the intersection of the altitude and the base.

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6-9 Practice

Form G

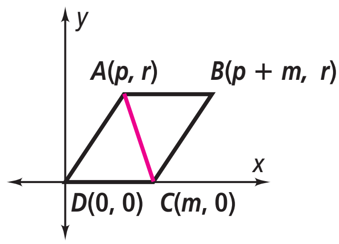
Proofs Using Coordinate Geometry

Use coordinate geometry to prove each statement. Follow the outlined steps.

1. Either diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Given: ▱*ABCD*

Prove: $\triangle ACD \cong \triangle CAB$



a. Use the figure at the right. Draw $\overline{AC}$.

b. Which theorem should you use to show that $\triangle ACD$ and $\triangle CAB$ are congruent? Explain.
SSS; side lengths can be shown to be equal using coordinate geometry.

c. Which formula(s) will you need to use?
Distance Formula

d. Show that $\triangle ACD$ and $\triangle CAB$ are congruent.
$AD = CB$. By the Distance Formula, both are equal to $\sqrt{p^2 + r^2}$. $AB = CD$. By the Distance Formula, both are equal to m. $AC = CA$ by the Reflexive Property of Equality. So, the triangles are congruent by SSS.

2. The diagonals of a parallelogram bisect one another.

Given: ▱*ABCD*

Prove: The midpoints of the diagonals are the same.

a. How will you place the parallelogram in the coordinate plane?
Answers may vary. Sample: with vertices $A(p, r)$; $B(p + m, r)$; $C(m, 0)$; and $D(0, 0)$

b. Find the midpoints of $\overline{AC}$ and $\overline{BD}$. What are the coordinates of the midpoints? **Answers may vary. Sample: The midpoint of AC is $\left(\frac{p+m}{2}, \frac{r}{2}\right)$; the midpoint of BD is $\left(\frac{p+m}{2}, \frac{r}{2}\right)$.**

c. Are the midpoints the same? Do the diagonals bisect one another?
yes; yes

d. Reasoning Would using a different parallelogram or labeling the vertices differently change your answer? Explain.
Answers may vary. Sample: No; the midpoints would still have identical coordinates.

3. How can you use coordinate geometry to prove that if the midpoints of a square are joined to form a quadrilateral, then the quadrilateral is a square? Explain.
Use the Midpoint Formula to find the midpoints of the square. Then use the Slope Formula to show that consecutive sides are perpendicular, and the Distance Formula to show that all sides are congruent.

6-9 Practice (continued) — Form G

Proofs Using Coordinate Geometry

Tell whether you can reach each conclusion below using coordinate methods. Give a reason for each answer.

4. A triangle is isosceles.
Yes; use the Distance Formula. You would need to prove that two sides of the triangle are congruent. You could do this by finding the distances between the points that form the triangle.

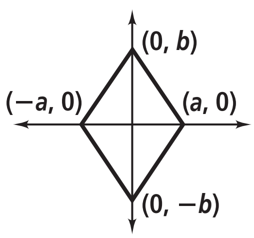
5. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.
Yes; find the midpoint of the hypotenuse by using the Midpoint Formula. Then find the distance of this midpoint from each vertex by using the Distance Formula.

6. If the midpoints of the sides of an isosceles trapezoid are connected, they will form a parallelogram.
Yes; find the midpoints of the sides by using the Midpoint Formula. Then use the Slope Formula to find the slopes of the segments formed by connecting these midpoints. If opposite sides are parallel, then the figure formed is a parallelogram.

7. The diagonals of a rhombus bisect one another.
Yes; use the Midpoint Formula to find the midpoint of the diagonals. If the midpoints are the same, then the diagonals bisect one another.

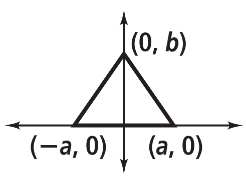
Use coordinate geometry to prove each statement.

8. The segments joining the midpoints of a rhombus form a rectangle.



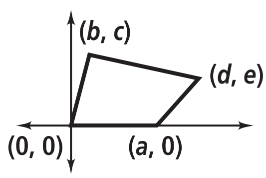
The midpoints are $(\frac{a}{2}, \frac{b}{2})$, $(-\frac{a}{2}, \frac{b}{2})$, $(-\frac{a}{2}, -\frac{b}{2})$, and $(\frac{a}{2}, -\frac{b}{2})$. The quadrilateral formed by these points has sides with vertical and horizontal slopes. Therefore, the consecutive sides are perpendicular, making the quadrilateral a rectangle.

9. The median to the base of an isosceles triangle is perpendicular to the base.



The median meets the base at (0, 0), the midpoint of the base. Therefore, the median has undefined slope, or is vertical. Because the base is a horizontal segment, the median is perpendicular to the base.

10. The segments joining the midpoints of a quadrilateral form a parallelogram.



The midpoints are $(\frac{a}{2}, 0)$, $(\frac{a+d}{2}, \frac{e}{2})$, $(\frac{b+d}{2}, \frac{c+e}{2})$, and $(\frac{b}{2}, \frac{c}{2})$. One pair of opposite sides has a slope of $\frac{e}{d}$, and the other pair has a slope of $\frac{c}{b-a}$. Therefore, it is a parallelogram.

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6-9 Practice

Form K

Proofs Using Coordinate Geometry

Developing Proof **Complete the following coordinate proofs.**

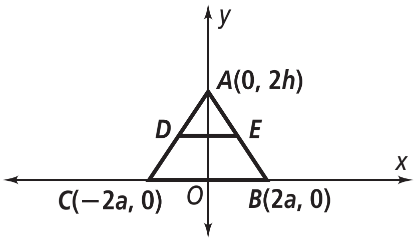
1. Triangle Midsegment Theorem

Given: $\triangle ABC$

D is the midpoint of $\overline{AC}$.

E is the midpoint of $\overline{AB}$.

Prove: $DE = \frac{1}{2}CB$ and $\overline{DE} \parallel \overline{CB}$.



a. Find the coordinates of D and E.

To start, use the Midpoint Formula.

$D\left(\frac{-2a + 0}{2}, \frac{2h + 0}{2}\right)$ or $D(\boxed{-a}, \boxed{h})$

$E\left(\frac{2a + 0}{2}, \frac{2h + 0}{2}\right)$ or $E(\boxed{a}, \boxed{h})$

b. Find DE and CB. **The length of the midsegment is half the length of the base.**

To start, use the Distance Formula.

$DE = \sqrt{(-a - a)^2 + (h - h)^2} = \boxed{2a}$

$CB = \sqrt{(-2a - 2a)^2 + (0 - 0)^2} = \boxed{4a}$

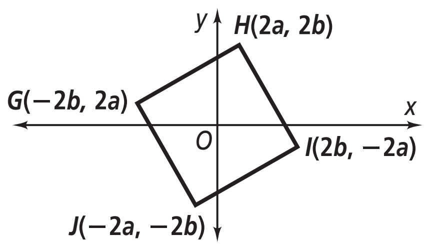
c. Find the slope of $\overline{DE}$ and the slope of $\overline{BC}$. Explain.
0; 0; the midsegment is ∥ to the base. The Triangle Midsegment Theorem is proved: The midsegment is ∥ to the base and half its length.

2. Reasoning In Exercise 1, explain why it is easier to use the coordinates $(0, 2h)$, $(2a, 0)$, and $(-2a, 0)$, rather than $(0, h)$, $(a, 0)$, and $(-a, 0)$.
When you use the Midpoint Formula to find the coordinates of *D* and *E*, you need to divide by 2. Using coordinates that are multiples of 2 makes this easier.

3. A parallelogram is a square.

Given: ▱ $GHIJ$

Prove: $GHIJ$ is a square.



a. Find GI and HJ. **$\sqrt{(4a)^2 + (4b)^2}$; $\sqrt{(4a)^2 + (4b)^2}$; The diagonals are ≅.**

b. Find the slopes of GI and HJ.
$-\frac{a}{b}$; $\frac{b}{a}$; the slopes are negative reciprocals, so the diagonals are ⊥. Therefore, *GHIJ* is a square because the diagonals are ≅ and ⊥.

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6-9 Practice (continued) — Form K

Proofs Using Coordinate Geometry

Tell whether you can reach each type of conclusion below using coordinate methods. Give a reason for each answer.

4. $m\angle A = m\angle G$

No; the Distance, Midpoint, and Slope Formulas do not provide information about angle measures unless additional information is given or can be determined.

5. $\triangle ABC \cong \triangle GHI$

Yes; use the Distance Formula to prove the △s are ≅ by SSS.

6. Quadrilateral *ABCD* is a rectangle.

Yes; answers may vary. Sample: Use the Slope Formula to show that consecutive sides are ⊥.

7. $\triangle HIJ$ is equiangular.

Yes; use the Distance Formula to prove the △ is equilateral.

8. The base angles of an isosceles trapezoid are congruent.

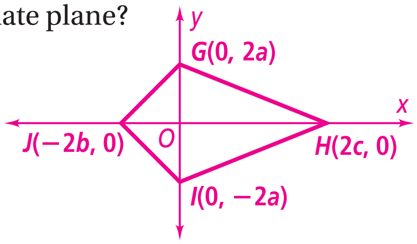
Yes; answers may vary. Sample: draw diagonals and use the Distance Formula to prove the triangles are congruent.

9. Quadrilateral *WXYZ* is a parallelogram.

Yes; answers may vary. Sample: use the Midpoint Formula to show that the diagonals bisect each other.

10. Think About a Plan If the diagonals of a quadrilateral are perpendicular but do not bisect each other, then the quadrilateral is a kite.

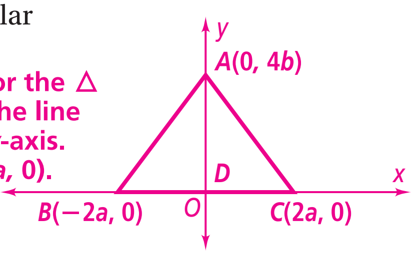
- How will you place the quadrilateral in the coordinate plane?
  so its diagonals are on the *x*- and *y*-axes
- What formula(s) will you use?
  Distance Formula
- What are the coordinates of the vertices?
  Answers may vary. Sample: $J(-2b, 0)$, $G(0, 2a)$, $H(2c, 0)$, $I(0, -2a)$



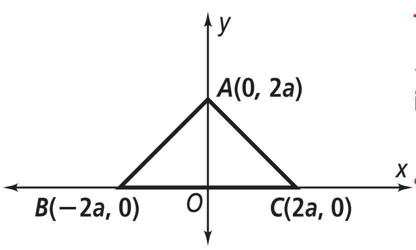
Use coordinate geometry to prove each statement.

11. The vertex of an isosceles triangle is on the perpendicular bisector of the base.

Place base of △ on *x*-axis with $A(-a, 0)$ and $B(a, 0)$. For the △ to be isosceles, *C* must be equidistant from *A* and *B*. The line containing all points equidistant from *A* and *B* is the *y*-axis. This line is the perpendicular bisector $A(-a, 0)$ and $B(a, 0)$.



12. $\triangle ABC$ is a right, isosceles triangle.



The slope of $\overline{AB} = \frac{2a - 0}{0 - (-2a)} = 1$ and the slope of $\overline{AC} = \frac{0 - 2a}{2a - 0} = -1$. Since the product of these slopes is -1, $\overline{AB} \perp \overline{AC}$, meaning that $\angle A$ is a right angle, so $\triangle ABC$ a right triangle; $AB = \sqrt{(2a)^2 + (-2a)^2} = \sqrt{8a^2}$ and $AC = \sqrt{(-2a)^2 + (2a)^2} = \sqrt{8a^2}$. Since $AB = AC$, $\triangle ABC$ is an isosceles triangle.

Name ______________________ Class ____________ Date ____________

6-9 Standardized Test Prep

Proofs Using Coordinate Geometry

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which of the following could you conclude using coordinate geometry? **A**

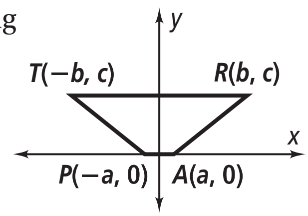
(A) $\triangle EFG$ is an equilateral triangle.

(B) $m\angle E = 60$

(C) $m\angle F = 99$

(D) $m\angle E = m\angle F$

2. Quadrilateral *TRAP* is shown at the right. Which of the following could you use to show that *TRAP* is a trapezoid? **H**



(F) Prove $RA = TP$.

(G) Prove $\overline{RA} \perp \overline{AP}$.

(H) Prove $\overline{TR} \parallel \overline{PA}$.

(I) Prove that there are no right angles formed by the line segments.

3. Which formula or formulas do you need to use to prove that if the segments connecting the midpoints of a trapezoid are joined they form a parallelogram? **D**

(A) Slope Formula

(B) Distance Formula

(C) Distance Formula and Slope Formula

(D) Slope Formula and Midpoint Formula

4. Which formula or formulas do you need to use to prove that a quadrilateral is an isosceles trapezoid? **H**

(F) Slope Formula

(G) Distance Formula

(H) Distance Formula and Slope Formula

(I) Slope Formula and Midpoint Formula

Short Response

5. How would you use coordinate geometry to prove that two line segments are perpendicular?

[2] Find the slopes of the line segments, and show that the product of the slopes is −1. [1] Find the slopes of the line segments. [0] no answer or wrong answer based on an inappropriate plan

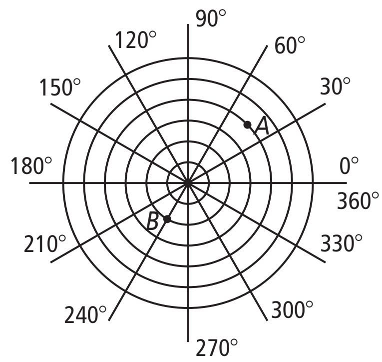
Name ______________________ Class ____________ Date ____________

6-9 Enrichment

Proofs Using Coordinate Geometry

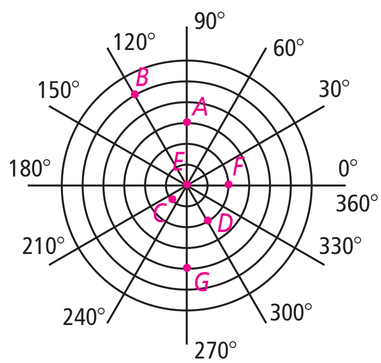
Another Type of Coordinate Plane

Another way to position a point in a plane is to use *polar coordinates.* A polar coordinate consists of two values separated by a semicolon, the *polar distance* and the *polar angle.* For example, $A(4; 45°)$ represents a point with a distance of 4 units from the origin or pole and a 45°angle measure from the positive x-axis. This point is graphed to the right on a polar graph. It is customary to measure angles counterclockwise from the positive x-axis. However, if the given angle measure is negative, then the angle is measured clockwise from the positive x-axis. For example, $B(2; -120°)$ would be graphed as shown on the diagram at the right.



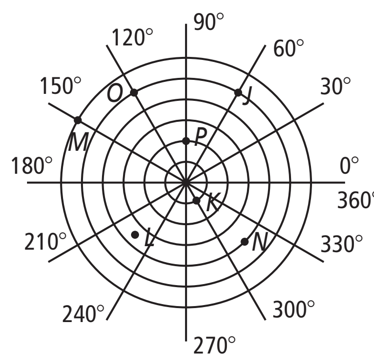
Graph and label the following points on the polar graph provided at the right.

1. $A(3; 90°)$
2. $B(5; 120°)$
3. $C(1; 225°)$
4. $D(2; -60°)$
5. $E(0; 330°)$
6. $F(2; 360°)$
7. $G(4; -90°)$



Use the polar graph at the right, and provide the polar coordinate for the indicated point on the graph.

8. J **(5; 60°)**
9. K **(1; 300°)**
10. L **(3.5; 225°)**
11. M **(6; 150°)**
12. N **(4; 315°)**
13. O **(5; 120°)**
14. P **(2; 90°)**



Graph each figure on a polar graph and provide coordinates for its vertices.
Check students' drawings.

15. isosceles triangle **Answers may vary. Sample: (2; 0°), (3; 120°), (3; 240°)**
16. rectangle **Answers may vary. Sample: (2; 30°), (2; 150°), (2; 210°), (2; 330°)**
17. trapezoid **Answers may vary. Sample: (2; 30°), (3; 120°), (3; 240°), (2; 330°)**

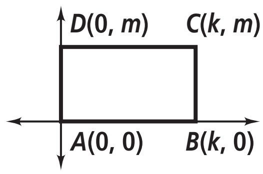
6-9 Reteaching

Proofs Using Coordinate Geometry

A *coordinate proof* can be used to prove geometric relationships. A coordinate proof uses variables to name coordinates of a figure on a coordinate plane.

Problem

Use coordinate geometry to prove that the diagonals of a rectangle are congruent.

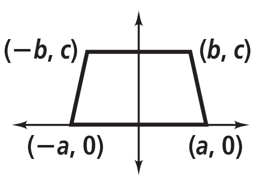


$$AC = \sqrt{(k-0)^2 + (m-0)^2}$$
$$= \sqrt{k^2 + m^2}$$
$$BD = \sqrt{(0-k)^2 + (m-0)^2}$$
$$= \sqrt{(-k)^2 + m^2}$$
$$= \sqrt{k^2 + m^2}$$
$$\overline{AC} \cong \overline{BD}$$

Exercises

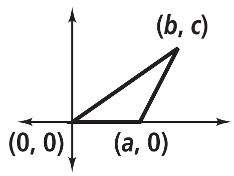
Use coordinate geometry to prove each statement.

1. Diagonals of an isosceles trapezoid are congruent.



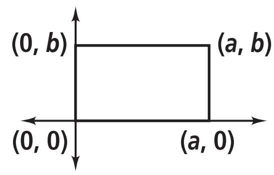
Use the Distance Formula to find the lengths of the diagonals. Each has a length of $\sqrt{c^2 + (b + a)^2}$. So, the diagonals are congruent.

2. The line containing the midpoints of two sides of a triangle is parallel to the third side.



You need to use the Midpoint Formula to find the midpoints of two sides, and the Slope Formula to show that the line connecting the midpoints is parallel to the third side. The midpoints are $(\frac{b}{2}, \frac{c}{2})$ and $(\frac{b+a}{2}, \frac{c}{2})$. The line connecting the midpoints has slope 0 and is therefore parallel to the third side.

3. The segments joining the midpoints of a rectangle form a rhombus.



You need to find the midpoints using the Midpoint Formula and lengths of segments connecting the midpoints with the Distance Formula. The midpoints are $(\frac{a}{2}, 0)$, $(a, \frac{b}{2})$, $(\frac{a}{2}, b)$, and $(0, \frac{b}{2})$. The segments joining these points are congruent, as they each have a length of $\frac{1}{2}\sqrt{a^2 + b^2}$.

6-9 Reteaching (continued)

Proofs Using Coordinate Geometry

The example used the Distance Formula to prove two line segments congruent. When planning a coordinate proof, write down the formulas that you will need to use, and write what you can prove using those formulas.

Exercises

State whether you can reach each conclusion below using coordinate methods. Give a reason for each answer.

4. $AB = \frac{1}{2}CD$.
Yes; use the Distance Formula to show that $AB = \frac{1}{2}CD$.

5. $\triangle ABC$ is equilateral.
Yes; use the Distance Formula to show all sides are equal.

6. Quadrilateral *ABCD* is a square.
Yes; use the Distance Formula to show all sides are equal and the Slope Formula to show all sides are perpendicular if the product of the slopes of any two adjacent sides is −1.

7. The diagonals of a quadrilateral form right angles.
Yes; use the Slope Formula to show the diagonals are ⊥ if the product of their slopes is −1.

8. Quadrilateral *ABCD* is a trapezoid.
Yes; use the Slope Formula to show that one pair of sides is parallel because the slopes are equal.

9. $\triangle ABC$ is a right triangle.
Yes; use the Slope Formula to show that the product of the slopes of two sides is −1.

10. Quadrilateral *ABCD* is a kite.
Yes; use the Distance Formula to check that there are two pairs of adjacent sides of the same length, and that all four sides are not the same length.

11. The diagonals of a quadrilateral form angles that measure 30 and 150.
No; I do not have coordinate methods to measure angles.

12. $m\angle D = 33$
No; I do not have coordinate methods to measure angles.

13. $\triangle ABC$ is scalene.
Yes; use the Distance Formula to show that all sides have a different length.

14. The segments joining midpoints of an equilateral triangle form an equilateral triangle.
Yes; use the Midpoint Formula to find the midpoints of the sides and the Distance Formula to show that the lengths of all three sides are equal in the new triangle.

15. Quadrilateral *KLMN* is an isosceles trapezoid.
Yes; use the Slope Formula to show that one pair of sides is parallel (have equal slopes), and use the Distance Formula to show that the legs are congruent.

Name ______________________ Class ____________ Date ____________

Chapter 6 Quiz 1

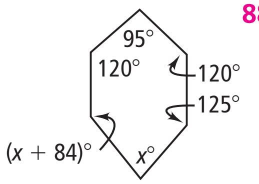
Form G

Lessons 6-1 through 6-6

Do you know HOW?

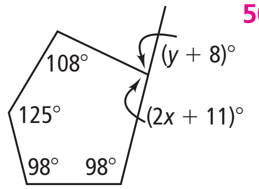
Find the value of each variable.

1.



88

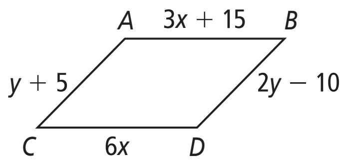
2.



50; 61

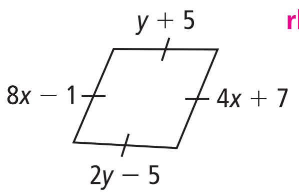
3. Find the value of each variable in parallelogram *ABCD*.

5; 15



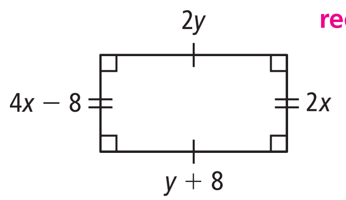
Classify the quadrilateral, then find the values of the variables.

4.



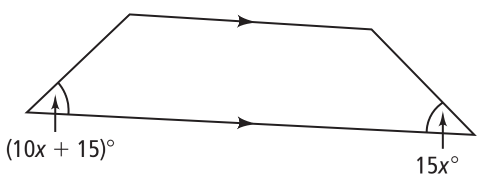
rhombus; 2; 10

5.



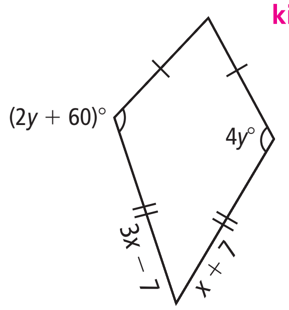
rectangle; 4; 8

6.



isosceles trapezoid; 3

7.



kite; 7; 30

Do you UNDERSTAND?

8. Vocabulary Is a rhombus with four congruent angles also a square? Explain.
Yes; a square is a rhombus that has four congruent angles.

9. Vocabulary If the diagonals of a quadrilateral are perpendicular, can the quadrilateral be a rectangle? Explain.
Yes; if it is a square, then the diagonals are perpendicular and all squares are rectangles.

10. Reasoning When you draw a diagonal of any parallelogram, what type of triangles do you get? Explain. Explanations may vary. Sample: The diagonal creates a common side for the two triangles and it is congruent to itself. The other two sides of each triangle are part of congruent pairs of sides on the parallelogram. By SSS, the triangles are congruent.

Name ______________________ Class ____________ Date ____________

Chapter 6 Quiz 2

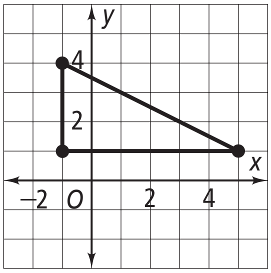
Form G

Lessons 6-7 through 6-9

Do you know HOW?

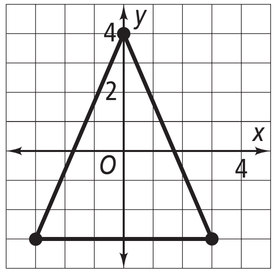
Determine whether each triangle is *scalene, isosceles,* or *equilateral.*

1.



scalene

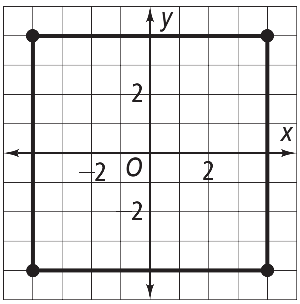
2.



isosceles

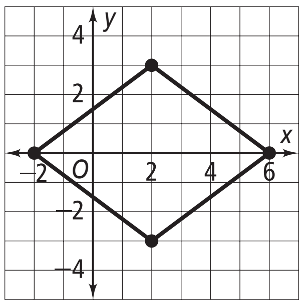
Determine whether the parallelogram is a *rhombus, rectangle, square,* or *none of these.*

3.



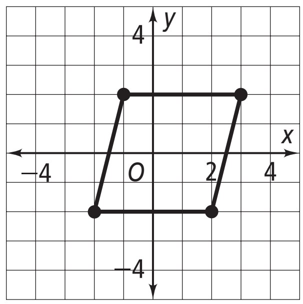
square

4.



rhombus

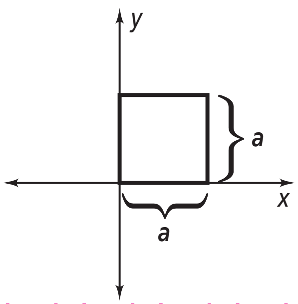
5.



none of these

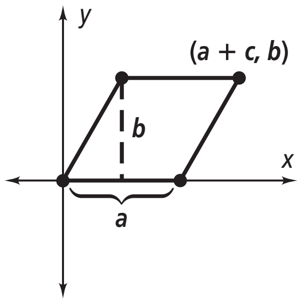
What are the coordinates of the vertices of each parallelogram?

6.



(0, 0); (*a*, 0); (*a*, *a*); (0, *a*)

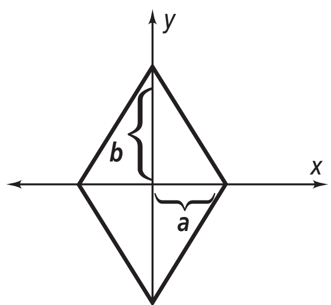
7.



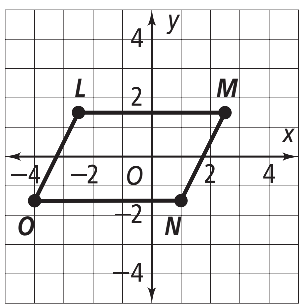
(0, 0); (*a*, 0); (*a* + *c*, *b*); (*c*, *b*)

8.

(*a*, 0); (0, *b*); (−*a*, 0); (0, −*b*)



Do you UNDERSTAND?



9. Reasoning Describe how you would determine *LMNO* is a parallelogram.

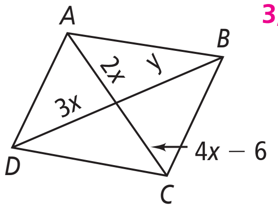
The slopes of $\overline{LM}$ and $\overline{ON}$ are both 0, so these segments are parallel. The slopes of lines $\overline{MN}$ and $\overline{LO}$ are both 2, so these segments are parallel. *LMNO* has two pairs of parallel sides, and is therefore a parallelogram.

Chapter 6 Test

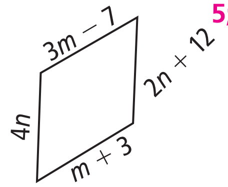
Form G

Do you know HOW?

Find the values of the variables in each parallelogram.

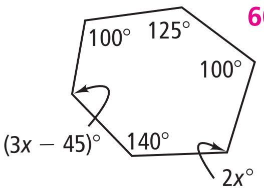
1.


3; 9

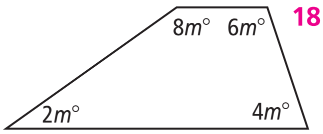
2.


5; 6

Find the values of the variable(s) in each figure.

3.


60

4.


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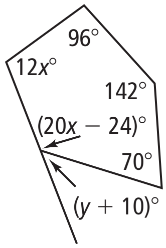
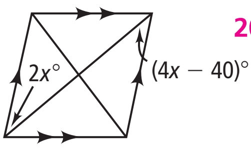
5.


8; 34

6.


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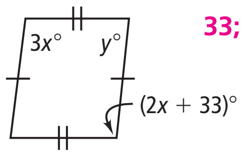
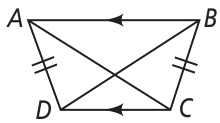
7.


33; 81

8. $AC = 7x - 15$, $BD = 4x + 15$



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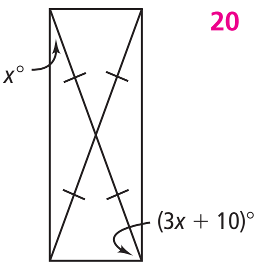
9.


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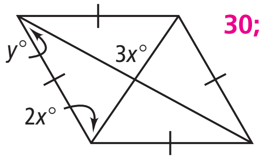
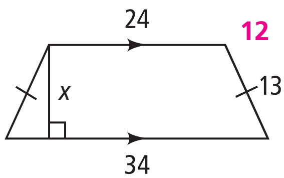
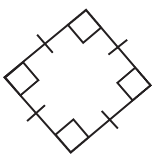
10.


30; 30

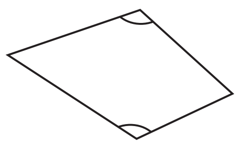
11.


12

Classify each figure as precisely as possible. Explain your reasoning.

12.


square; four congruent sides and four congruent angles

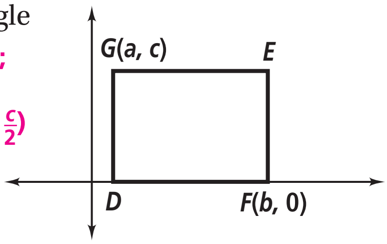
13.


quadrilateral; only one pair of opposite angles congruent

Give the coordinates for points D and E without using any new variables. Then find the midpoint of $\overline{DE}$.

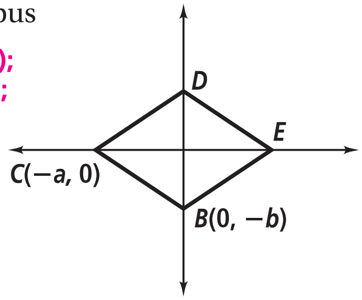
14. rectangle

$D(a, 0)$; $E(b, c)$; $\left(\frac{a+b}{2}, \frac{c}{2}\right)$



15. rhombus

$D(0, b)$; $E(a, 0)$; $\left(\frac{a}{2}, \frac{b}{2}\right)$



Do You UNDERSTAND?

Reasoning **Determine whether each statement is true or false. If true, explain your reasoning. If false, provide a counterexample.**

16. The diagonals of a rectangle always form four congruent triangles.

False; the diagonals form two pairs of congruent triangles.

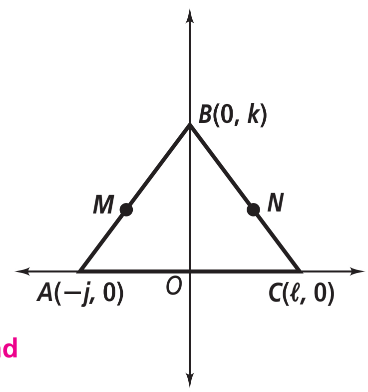
17. If the diagonals of a quadrilateral are perpendicular, then the quadrilateral must be a kite.

False; squares and rhombuses also have perpendicular diagonals.

18. Reasoning Use coordinate geometry to prove the following:

Given: $\triangle ABC$ with vertices $(-j, 0)$, $(0, k)$, $(\ell, 0)$, and midpoints M, N, and O of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$

Prove: The perimeter of $\triangle MNO$ is one-half the perimeter of $\triangle ABC$.



$BC = \sqrt{k^2 + \ell^2}$, $AB = \sqrt{j^2 + k^2}$, $AC = \ell + j$. Therefore, the perimeter of $\triangle ABC$ is $\ell + j + \sqrt{j^2 + k^2} + \sqrt{k^2 + \ell^2}$. The coordinates of the midpoints are: $M\left(-\frac{j}{2}, \frac{k}{2}\right)$, $N\left(\frac{\ell}{2}, \frac{k}{2}\right)$, and $O\left(\frac{\ell - j}{2}, 0\right)$. $MN = \frac{1}{2}(\ell + j)$, $NO = \frac{1}{2}\sqrt{j^2 + k^2}$, and $MO = \frac{1}{2}\sqrt{k^2 + \ell^2}$. So, the perimeter of $\triangle MNO = \frac{1}{2}\left(\ell + j + \sqrt{j^2 + k^2} + \sqrt{k^2 + \ell^2}\right)$, or half the perimeter of $\triangle ABC$.

19. Compare and Contrast Explain in what ways a rectangle is similar to and different from a kite.

A rectangle and a kite are both quadrilaterals with two pairs of sides congruent. However, a rectangle has congruent opposite sides. A kite has two pairs of consecutive congruent sides, and no opposite sides congruent.

Name ______________________ Class ____________ Date ____________

Chapter 6 Quiz 1

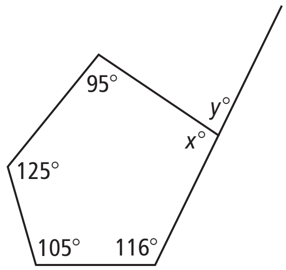
Form K

Lessons 6-1 through 6-6

Do you know HOW?

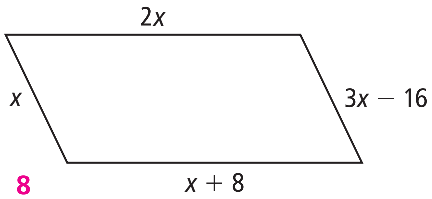
1. What is the sum of the interior angle measures of a 15-gon? **2340°**

2. Find the values of the variables in the figure at the right. **99; 81**



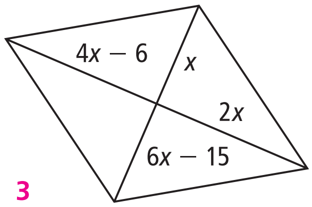
Find the value of *x* for which *ABCD* is a parallelogram.

3.



8

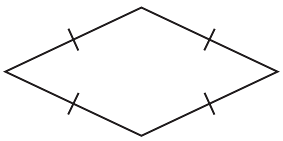
4.



3

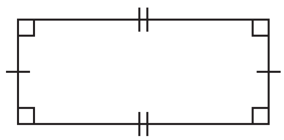
Classify each quadrilateral as precisely as possible.

5.



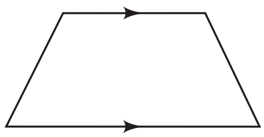
rhombus

6.



rectangle

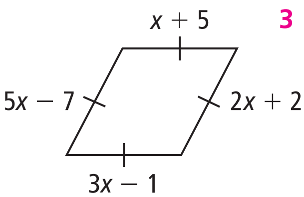
7.



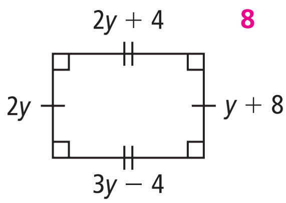
trapezoid

Find the value of the variables for each figure.

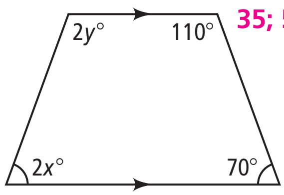
8. **3**



9. **8**



10. **35; 55**



Do you UNDERSTAND?

11. How can you classify a rhombus with four congruent angles? Explain.
Square; if the ⦞ are ≅, then each measures 90. A quadrilateral with four ≅ sides and four 90° ⦞ is a square.

12. Reasoning Explain why drawing a diagonal on any parallelogram will always result in two congruent triangles.
Answers may vary. Sample: The diagonal makes a common side for the two triangles that is congruent to itself by the Refl. Prop. of ≅. The other two sides of each △ are part of ≅ opposite sides of the parallelogram. The ⦞ are therefore ≅ by SSS.

Name ______________________ Class ____________ Date ____________

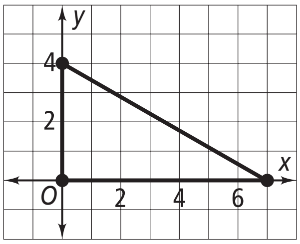
Chapter 6 Quiz 2

Form K

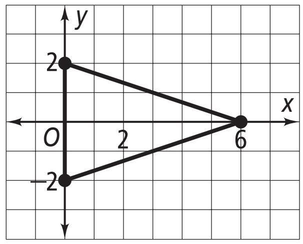
Lessons 6-7 through 6-9

Do you know HOW?

Determine whether each triangle is *isosceles, scalene,* or *equilateral.*

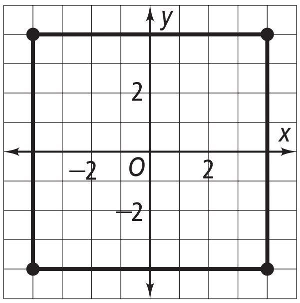
1. 

scalene

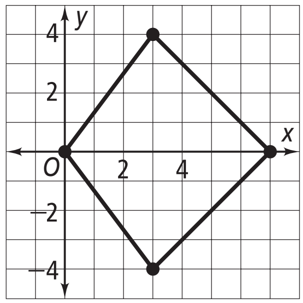
2. 

isosceles

Determine whether the figure is a *rhombus, rectangle, square* or *none.*

3. 

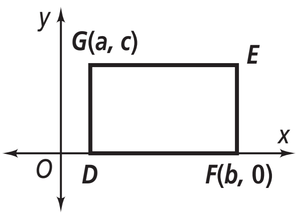
square

4. 

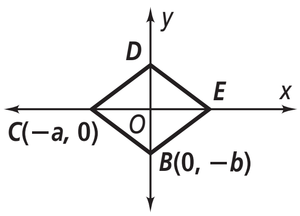
none

Give the coordinates for *D* and *E* that make the figure indicated, without using any new variables.

5. rectangle **$D(a, 0)$; $E(b, c)$**



6. rhombus **$D(0, b)$, $E(a, 0)$**



Do you UNDERSTAND?

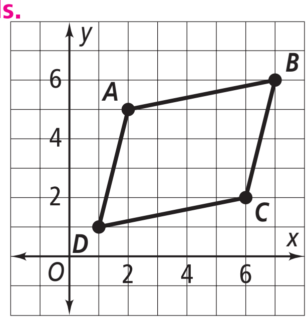
7. Reasoning A quadrilateral has perpendicular diagonals. Is this enough to determine what type of quadrilateral it is? Explain. **No; explanations may vary. Sample: a square, a kite, and a rhombus all have perpendicular diagonals.**

8. Reasoning Use coordinate geometry to prove the following:

Given: ▱*ABCD* with vertices at $A(2, 5)$, $B(7, 6)$, $C(6, 2)$, and $D(1, 1)$

Prove: ▱*ABCD* is a parallelogram.

Because $\overline{AB}$ and $\overline{DC}$ both have slopes of $\frac{1}{5}$, $\overline{AB}$ and $\overline{DC}$ are parallel. Because $\overline{AD}$ and $\overline{BC}$ both have slopes of 4, $\overline{AD}$ and $\overline{BC}$ are parallel. ▱*ABCD* has two pairs of parallel sides. Therefore, *ABCD* is a parallelogram.



Name ______________________ Class ____________ Date ____________

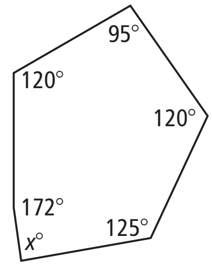
Chapter 6 Test

Form K

Do you know HOW?

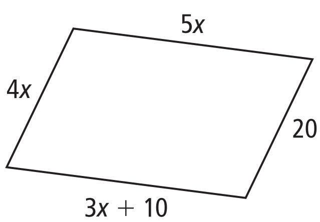
1. Find the sum of the angle measures of an 11-gon. **1620°**

2. Find the value of x in the figure at the right. **88**

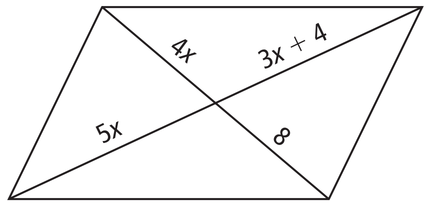


Find the value of x for which $ABCD$ is a parallelogram.

3. **5**

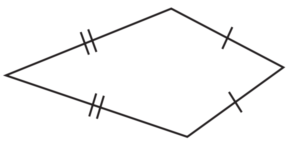


4. **2**



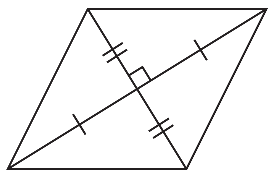
Classify each quadrilateral as precisely as possible.

5.



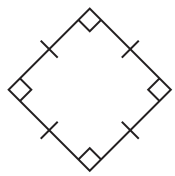
kite

6.



rhombus

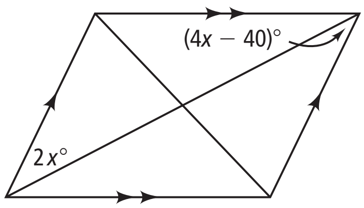
7.



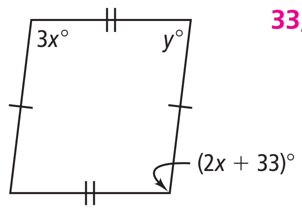
square

Find the values of the variables in each figure.

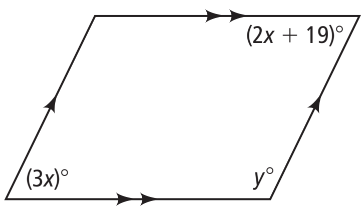
8. **20**



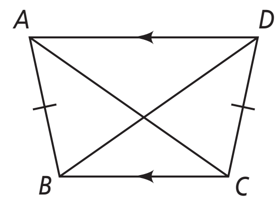
9. **33; 81**



10. **19; 123**

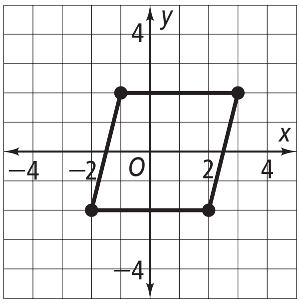


11. $AC = 5z - 7$, $BD = 2z + 11$ **6**



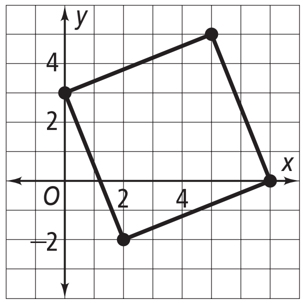
Determine whether the parallelogram is a *rhombus, rectangle, square* or *none*.

12.



none

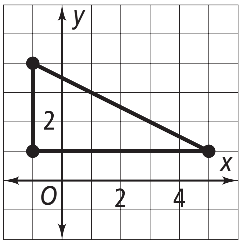
13.



square

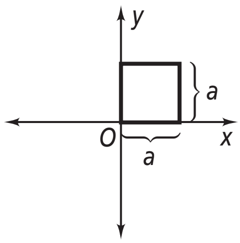
What are the coordinates for the vertices of each figure?

14. right triangle



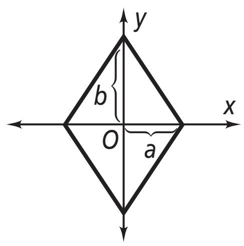
(−1, 1), (−1, 4), (5, 1)

15. square



(0, 0), (a, 0), (0, a), (a, a)

16. rhombus



(a, 0), (0, b), (−a, 0), (0, −b)

Do you UNDERSTAND?

Reasoning **Determine whether each statement is *true* or *false*. Explain your reasoning.**

17. A graph of a trapezoid can have diagonals with slopes that are negative reciprocals and two pairs of adjacent sides that are congruent.
False; the description matches a kite or a rhombus; it cannot be a trapezoid because either two pairs or no pairs of opposite sides are parallel.

18. A trapezoid has congruent diagonals and one set of opposite sides that have equal slopes.
False; trapezoids do not always have congruent diagonals.

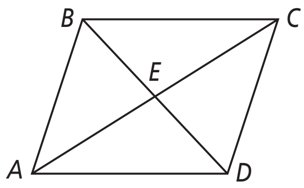
19. A figure that has opposite sides with equal length and equal slopes and diagonals with slopes that are negative reciprocals must be a rhombus. True; any rhombus fits this description.

Name ______________________________ Class ____________ Date ____________

Performance Tasks

Chapter 6

Task 1



a. If $ABCD$ is a parallelogram, list everything you know about the sides and angles in the figure.

b. Which of the following conditions are sufficient, individually, to guarantee that $ABCD$ is a parallelogram?

A. $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

B. $\overline{BD} \perp \overline{AC}$

C. $\overline{BD}$ and $\overline{AC}$ bisect each other.

D. $\overline{BE} \cong \overline{EC}$ and $\overline{AE} \cong \overline{ED}$

E. $\angle BAD \cong \angle BCD$ and $\angle ABC \cong \angle ADC$

F. $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

Task 1: Scoring Guide

Samples:

a. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$, $\angle ABD \cong \angle CDB$, $\angle CBD \cong \angle ADB$, $\angle ACD \cong \angle CAB$, $\angle ACB \cong \angle CAD$, $BE = ED$, $AE = EC$, $\angle ABC \cong \angle CDA$, $\angle BCD \cong \angle BAD$

b. C, E, F

[4] Student lists all statements accurately in part (a) and gives the correct answer in part (b).

[3] Student gives mostly correct answers but with some errors.

[2] Student gives some correct answers, but is missing many of the correct answers.

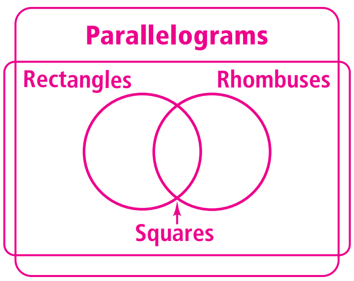
[1] Student gives answers that fail to demonstrate understanding of the properties of parallelograms.

[0] Student makes little or no effort.

Task 2

Draw a Venn diagram showing the relationships among squares, rectangles, rhombuses, and parallelograms.

Task 2: Scoring Guide



[4] Student gives accurate and complete answers and diagram.

[3] Student gives answers and a diagram that are mostly accurate.

[2] Student draws a diagram that is somewhat accurate.

[1] Student gives answers or a diagram containing significant errors.

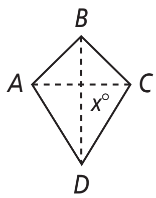
[0] Student makes little or no effort.

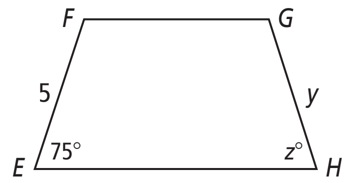
Performance Tasks (continued)

Chapter 6

Task 3

ABCD is a kite, and *EFGH* is an isosceles trapezoid with $\overline{FG} \parallel \overline{EH}$. Find the value of each variable, and support each answer by stating a definition or theorem.





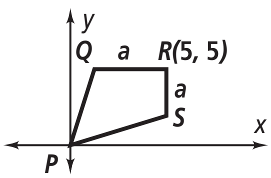
Task 3: Scoring Guide
$x = 90$ (Diagonals of a kite are $\perp$.)
$y = 5$ (Def. of isosc. trapezoid)
$z = 75$ (Base angles of isosc. trap. are $\cong$.)
[4] Student gives correct answers and reasons.
[3] Student gives correct answers and some correct reasons.
[2] Student gives mostly correct answers and reasons.
[1] Student gives mostly incorrect answers and reasons.
[0] Student makes little or no effort.

Task 4

PQRS is a kite, with $QR = RS = a$, where $0 < a < 5$.

a. Give the coordinates of *Q* and *S* in terms of *a*.

b. Prove that $\overline{PR} \perp \overline{QS}$.



Task 4: Scoring Guide
a. $Q = (5 - a, 5)$; $S = (5, 5 - a)$
b. Slope of $\overline{PR} = \frac{5-0}{5-0} = 1$. Slope of $\overline{QS} = \frac{5-a-5}{5-(5-a)} = -1$. Because the product of their slopes $= -1$, $\overline{PR} \perp \overline{QS}$.
[4] Student gives correct coordinates and a valid proof.
[3] Student gives correct coordinates and some correct reasoning.
[2] Student gives answers or a proof containing minor errors.
[1] Student gives incorrect coordinates in part (a) or a poorly constructed proof in part (b).
[0] Student makes little or no effort.

Name ______________________ Class ____________ Date ____________

Cumulative Review

Chapters 1–6

Multiple Choice

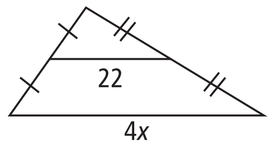
1. Which line is perpendicular to $y = \frac{1}{3}x + 9$? C

(A) $4y = 12x - 7$ (B) $3y = -6x + 11$ (C) $3y = -9x + 1$ (D) $6y = 2x - 1$

2. Which side lengths would not make a triangle? F

(F) 4, 3, 7 (G) 3, 8, 6 (H) 2, 4, 5 (I) 4, 6, 9

3. What is the value of x in the figure at the right? B



(A) 5.5 (C) 22

(B) 11 (D) 44

4. A circular copper decoration has a diameter of 2 in. How much more area would a circular decoration with a diameter of 6 in. have? F

(F) 8π in.^{2} (G) 10π in.^{2} (H) 32π in.^{2} (I) 40π in.^{2}

5. What is the inverse of the statement, "If the sky is blue, then it is not raining"? A

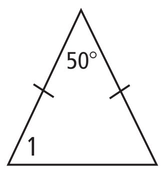
(A) If the sky is not blue, then it is raining.

(B) If it is not raining, then the sky is blue.

(C) If it is raining, then the sky is not blue.

(D) If the sky is blue, then it is raining.

6. What is $m\angle 1$ in the triangle at the right? I



(F) 50 (H) 60

(G) 55 (I) 65

7. Which conditions are sufficient to prove a quadrilateral is a square? B

I. All four sides are congruent.
II. The diagonals are congruent.
III. The diagonals bisect each other.

(A) I only (B) I and II (C) I and III (D) II and III

8. An isosceles triangle has two angles measuring 55° and 70°. What is the measure of the third angle? G

(F) 15 (G) 55 (H) 70 (I) 110

Cumulative Review (continued)

Chapters 1–6

Short Response

For Exercises 9–15, explain how you got your answer.

9. What is the length of $\overline{MN}$ if $LN = 42$?

[2] 18; $3x + 2x + 2 = 42$ by Segment Addition. So, $x = 8$ and $2(8) + 2 = 18$. [1] correct answer or explanation [0] incorrect answer with poor explanation

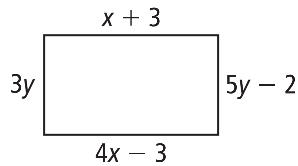
10. What is the midpoint of a segment with endpoints at $(-4, 3)$ and $(2, 5)$?

[2] $(-1, 4)$; $\left(\frac{-4 + 2}{2}, \frac{3 + 5}{2}\right) = (-1, 4)$ [1] correct answer or correct explanation [0] incorrect answer with poor explanation

11. What is the slope of a line that passes through at $(-1, 5)$ and $(4, 5)$?

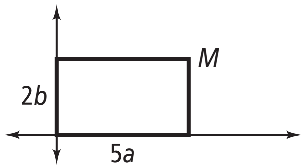
[2] 0; $\frac{5 - 5}{4 - (-1)} = \frac{0}{5}$ [1] correct answer or correct explanation [0] incorrect answer with poor explanation

12. What are the values of x and y for which quadrilateral $ABCD$ is a parallelogram? Explain.



[2] 2; 1; $x + 3 = 4x - 3$, $3y = 5y - 2$ [1] correct answer or correct explanation [0] incorrect answer with poor explanation

13. What are the coordinates of point M?



[2] $(5a, 2b)$; M is $5a$ to the right of the origin and $2b$ up from the origin. [1] correct answer or correct explanation [0] incorrect answer with poor explanation

14. What is the measure of an interior angle of a regular octagon? Explain.

[2] 135°; $\frac{(8 - 2)180}{8} = 135$ [1] correct answer or correct explanation [0] incorrect answer with poor explanation

15. What condition is necessary to assure that rhombus $LMNO$ is a square?

[2] All angles must be congruent, because a square is a rhombus with four right angles. [1] correct answer or correct explanation [0] incorrect answer with poor explanation

Extended Response

16. Sketch and label rhombus $ABCD$ and draw diagonal $\overline{AC}$.
Write a plan for proving $\triangle ABC \cong \triangle CDA$. **Check students' work. [4] Student draws and correctly labels the rhombus. Student writes a proof plan that uses SSS to prove the two ▵ ≅. [3] Student writes a proof plan but leaves out one critical point. [2] Student draws and labels the rhombus incorrectly or writes a proof plan that is partially complete. [1] Student draws and labels the rhombus correctly but does not give any proof plan. [0] incorrect answer with poor explanation**

Chapter 6 Project Teacher Notes: Go Fly a Kite

About the Project

Students will make a fully functioning kite using a sheet of paper and two staples. Then, they will explore how the weight and form of a kite affect its ability to fly. Finally, students will design, build, and fly their own kites.

Introducing the Project

Ask students to share their experiences with kites. Ask them to describe the shapes and features of kites they have flown. Have them share techniques they used in flying them.

Activity 1: Doing

To save time, have the supplies (paper, ruler, stapler, and thread) ready before students begin this activity. Read the directions aloud with students, and make sure they understand each step. You may want to begin by demonstrating the steps. Show students how to use the edge of a ruler to make a crease.

Activity 2: Analyzing

Make sure students understand *effective area*. Figures 1 and 2 show only the top part of the kite. Vertical sticks extend to the bottom part, which is congruent to the top. If possible, use a box to demonstrate. Students must know the properties of a rhombus in order to complete this activity.

Activity 3: Researching

Check with your school or local library for books they have on hand or can borrow about kites. You may be wise to make some of these books available in the classroom.

Finishing the Project

You might plan a project day on which students share their completed projects. Encourage students to share their processes as well as their products. Have students review their kite designs. Ask them to share the different models they tried and how each model or design performed. Ask students to share any insights they had when designing their kites, such as what made their kites perform better and what did not work.

Name ______________________________ Class ____________ Date ____________

Chapter 6 Project: Go Fly a Kite

Beginning the Chapter Project

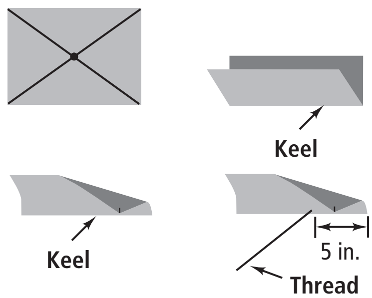
If you think kites are mere child's play, think again. From ancient China to modern times, geometric arrangements of fabric and rods have helped people rescue sailors, vanquish enemies, predict the weather, invent the airplane, study wind power, and entertain themselves with displays of aerodynamic artistry.

In your project for this chapter, you will turn a sheet of paper and a couple of staples into a fully functioning kite. You will explore how weight and form determine whether a kite sinks or soars. Finally, you will design, build, and fly your own kite. You will see how geometry can make a kite light and strong—spelling the difference between flight and failure.

Activities

Activity 1: Doing

Keel Kite



Building a kite usually takes a lot of time, patience, and care. You can build a simple paper kite by following these directions.

Draw diagonals to find the center of an $8\frac{1}{2}$-in.-by-11-in. sheet of paper. Then fold the paper, creasing only about a half inch past the center, to form the keel.

Bend each of the front corners of the paper out over to the keel. With a single staple, attach both corners to the keel and to each other about 1 in. from the front of the kite.

Attach one staple perpendicular to the keel as shown, and tie one end of a spool of thread to it. Your kite is ready to fly.

Write a paragraph that describes how your kite flew, and any modifications you might make to your kite to make it fly better. **Check students' work.**

Activity 2: Analyzing

Box Kite

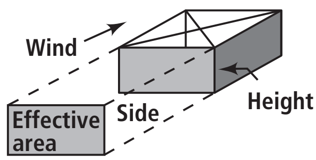


Figure 1

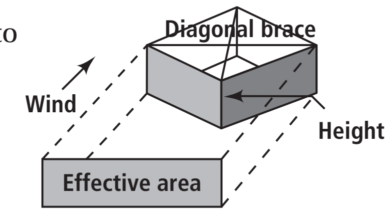


Figure 2

Weight and area exposed to the wind are key factors in kite design. The greater the *effective area* facing the wind and the lighter the kite, the less wind you need to get the kite off the ground.

1. In Figure 1, a face of the square box kite is perpendicular to the wind. Describe the shape of the effective area.
   The effective area is rectangular.
2. In Figure 2, a diagonal brace of the square box is perpendicular to the wind. Describe the shape of the effective area and the difference between this effective area and the effective area in Figure 1.
   The effective area is rectangular. The effective area is larger in Figure 2 than in Figure 1 because the diagonal is longer than the face.

Chapter 6 Project: Go Fly a Kite (continued)

3. Where on a box kite would you tie the string to get the greatest effective area? Explain.
   You should tie the string to a vertical stick of the kite.

4. If you use two different lengths of wood for the diagonal braces of a box kite, you can make a rhomboid box kite. Explain why changing a square box kite to a rhomboid box kite can increase the effective area.
   If the faces of the kite were unchanged, one diagonal of the rhombus is longer than the diagonals of the square, so the effective area would increase.

Activity 3: Researching

To help you decide what kind of kite you will design and build for this project, research at least three different kinds of kites. Books, magazines, and the Internet are good sources of information. Summarize your research. Include the following in your report:

- a description and sketch of each kite
- a list of materials needed to build each kite
- the wind conditions for which each kite is suited
  Check students' work.

Finishing the Project

Design and construct a kite, and then fly it. Include complete plans for building the kite, specifying the size, shape, and material for each piece. Use geometric terms in your plans. Experiment with different designs and models. Describe how your model performs and what you would do to improve it.

Reflect and Revise

Ask a classmate to review your project with you. Together, check that the diagrams and explanations are clear, complete, and accurate. Have you tried several designs and kept a record of what you learned from each? Could your kite be stronger, more efficient, or more pleasing to the eye? Revise your work as needed.

Extending the Project

Research the history of kites. Try to find examples of each use mentioned in the opening description of the project. You might make an illustrated time line to show special events, such as Benjamin Franklin's discovery about lightning and electricity and Samuel Cody's man-lifting kite.

Name ______________________________________ Class _______________ Date ______________

Chapter 6 Project Manager: Go Fly a Kite

Getting Started

As you work on the project, you will need a spool of thread, a stapler, and whatever materials you choose for building your own kite (such as straws, wooden dowels, fiberglass rods, plastic tubing, plastic bags, fabric, tissue paper, crepe paper, strapping tape, sticky tape, fishing line, kite string, glue).
Keep all your work for the project in a folder, along with this Project Manager.

Checklist

- ❑ Activity 1: paper kite
- ❑ Activity 2: box kite
- ❑ Activity 3: researching kite types
- ❑ your own kite

Suggestions

- ❑ You may wish to use colored paper or decorate your kite with drawings.
- ❑ Figures 1 and 2 show only the top part of the kite. Vertical sticks extend to the bottom part, which is just like the top.
- ❑ Find out whether there is a local kite-flying club that you can contact.
- ❑ Kite books offer construction plans and tips on building and flying kites. Start with a simple kite, and then modify the design as you become more confident. Don't be afraid to make mistakes.

Scoring Rubric

4 Your kite shows a great deal of effort. The diagrams and explanations in your plans and answers are clear, complete, and accurate. You use geometric language appropriately and correctly. You give a complete account of your experiments and how they led to improved designs.

3 Your kite shows reasonable effort. Your kite plans and your answers to questions are mostly understandable but may contain some minor errors and omissions. Most of the geometric language is used appropriately and correctly. You give some indication of why you designed your kite the way you did.

2 Your kite shows little effort. Diagrams and explanations are hard to follow or misleading. Geometric terms are not used, used sparsely, or often misused. You offer few or no reasons for your design.

1 Major elements of the project are incomplete or missing

0 Project not handed in or shows no effort.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric*.

Teacher's Evaluation of Project