HIGH SCHOOL GRADES 9-12

Number and Operations

Students in the secondary years extend their understanding of numbers to include the complex numbers and become proficient with advanced numeric relationships, including exponential, logarithmic, trigonometric, and matrix expressions. Students develop fluency in operating with and evaluating algebraic expressions.

Geometry and Measurement

Appropriate tools are used to create figures and identify characteristics and properties that describe relationships among shapes. Students use and apply algebraic representations to describe geometric relationships and transformations. Basic trigonometric relationships and functions are developed to solve problems. Students will move from an inductive approach to deductive methods of proof in their study of geometric figures. A variety of proof strategies is used to verify generalizations and properties of shape and connect geometry to other branches of mathematics.

Students maintain and extend measurement skills from earlier years, especially through science and technical course work. They focus on solving measurement-related problems involving concepts of precision, tolerance, error, and multiple dimensions.

Data Analysis and Probability

Students use appropriate technology to investigate, analyze, and present data. Measures of central tendency, spread, and distribution are used to describe and analyze data. Hypotheses, arguments, and conclusions are made, tested, and evaluated based on data. Using a best-fit function for bivariate data, students interpret and apply the function in the context of a problem.

Building upon study in earlier years, students use theoretical and experimental probability to model and solve increasingly complex problems.

Algebra   Students will be expected to describe and translate among graphic, algebraic, numeric, and verbal representations of relations and use those representations to solve problems. They use symbols to represent variables, parameters, and functions and extend their use of symbols to include vectors and matrices. Students use technology to assist in developing models and analytical solutions. They use appropriate terminology and notation to define function, domain, range, composition, and inverses of functions. They expand their understanding of functions to include power, polynomial, exponential, periodic, piece-wise, and recursively defined functions. They solve equations, inequalities, and systems using algebraic, tabular, numerical, and graphical methods.

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Introductory Mathematics

Introductory Mathematics provides students a survey of preparatory topics for high school mathematics, including the foundations for high school algebra and geometry. Appropriate technology, from manipulatives to calculators, should be used regularly for instruction and assessment.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will understand and compute with real numbers.

Objectives

1.01 Develop number sense for the real numbers.
   a) Define and use irrational numbers.
   b) Compare and order.
   c) Use estimates of irrational numbers in appropriate situations.

1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

COMPETENCY GOAL 2: The learner will use properties and relationships in geometry and measurement concepts to solve problems.

Objectives

2.01 Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.

2.02 Apply and use concepts of indirect measurement.

2.03 Represent problem situations with geometric models.

2.04 Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.

2.05 Identify, predict, and describe dilations in the coordinate plane.

COMPETENCY GOAL 3: The learner will understand and use graphs and data analysis.

Objectives
3.01   Collect, organize, analyze, and display data (including scatterplots) to solve problems.
3.02   Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.
3.03   Identify misuses of statistical and numerical data.

COMPETENCY GOAL 4: The learner will understand and use linear relations and functions.

Objectives
4.01   Develop an understanding of function.
   a) Translate among verbal, tabular, graphic, and algebraic representations of functions.
   b) Identify relations and functions as linear or nonlinear.
   c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.
   d) Interpret and compare properties of linear functions from tables, graphs, or equations.
4.02   Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.
4.03   Solve problems using linear equations and inequalities; justify symbolically and graphically.
4.04   Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

Algebra 1

Algebra 1 continues the study of algebraic concepts. It includes operations with polynomials and matrices, creation and application of linear functions and relations, algebraic representations of geometric relationships, and an introduction to nonlinear functions. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Operate with the real numbers to solve problems.
- Find, identify, and interpret the slope and intercepts of a linear relation.
- Visually determine a line of best fit for a given scatterplot; explain the meaning of the line; and make predictions using the line.
- Collect, organize, analyze, and display data to solve problems.
- Apply the Pythagorean Theorem to solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will perform operations with numbers and expressions to solve problems.

Objectives
1.01 Write equivalent forms of algebraic expressions to solve problems.
    a) Apply the laws of exponents.
    b) Operate with polynomials.
    c) Factor polynomials.
1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03 Model and solve problems using direct variation.

COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.

Objectives
2.01   Find the lengths and midpoints of segments to solve problems.
2.02   Use the parallelism or perpendicularity of lines and segments to solve problems.

COMPETENCY GOAL 3: The learner will collect, organize, and interpret data with matrices and linear models to solve problems.

Objectives
3.01   Use matrices to display and interpret data.
3.02   Operate (addition, subtraction, scalar multiplication) with matrices to solve problems.
3.03   Create linear models for sets of data to solve problems.
    a) Interpret constants and coefficients in the context of the data.
    b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.

Objectives
4.01   Use linear functions or inequalities to model and solve problems; justify results.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret constants and coefficients in the context of the problem.
4.02   Graph, factor, and evaluate quadratic functions to solve problems.
4.03   Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.
4.04   Graph and evaluate exponential functions to solve problems.

Geometry

Geometry continues students' study of geometric concepts building upon middle school topics. Students will move from an inductive approach to deductive methods of proof in their study of two- and three-dimensional geometric figures. Reasoning skills will be emphasized and students will broaden their use of the coordinate plane. Appropriate technology, from manipulatives to calculators and graphics software, should be used regularly for instruction and assessment.

Prerequisites
- Apply geometric properties and relationships to solve problems.
- Use formulas to solve problems.
- Define and use linear expressions to model and solve problems.
- Operate with matrices to model and solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will perform operations with real numbers to solve problems.

    Objectives
    1.01   Use the trigonometric ratios to model and solve problems involving right triangles.
    1.02   Use length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
    1.03   Use length, area, and volume to model and solve problems involving probability.

COMPETENCY GOAL 2: The learner will use geometric and algebraic properties of figures to solve problems and write proofs.

Objectives
2.01   Use logic and deductive reasoning to draw conclusions and solve problems.

2.02   Apply properties, definitions, and theorems of angles and lines to solve problems and write proofs.

2.03   Apply properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs:
   a) Triangles.
   b) Quadrilaterals.
   c) Other polygons.
   d) Circles.

2.04   Develop and apply properties of solids to solve problems.

COMPETENCY GOAL 3: The learner will transform geometric figures in the coordinate plane algebraically.

Objectives
3.01   Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

3.02   Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

Algebra 2

Algebra 2 continues students' study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Emphasis should be placed on practical applications and modeling. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Operate with matrices to solve problems.
- Create linear models, for sets of data, to solve problems.
- Use linear functions and inequalities to model and solve problems.
- Use quadratic functions to model problems and solve by factoring and graphing.
- Use systems of linear equations or inequalities to model and solve problems.
- Graph and evaluate exponential functions to solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will perform operations with complex numbers, matrices, and polynomials.

Objectives
1.01   Simplify and perform operations with rational exponents and logarithms (common and natural) to solve problems.
1.02   Define and compute with complex numbers.
1.03   Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.
1.04   Operate with matrices to model and solve problems.
1.05   Model and solve problems using direct, inverse, combined and joint variation.

COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.

Objectives

2.01   Use the composition and inverse of functions to model and solve problems; justify results.

2.02   Use quadratic functions and inequalities to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the constants and coefficients in the context of the problem.

2.03   Use exponential functions to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the constants, coefficients, and bases in the context of the problem.

2.04   Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data.
  a) Interpret the constants, coefficients, and bases in the context of the data.
  b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

2.05   Use rational equations to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the constants and coefficients in the context of the problem.
  c) Identify the asymptotes and intercepts graphically and algebraically.

2.06   Use cubic equations to model and solve problems.
  a) Solve using tables and graphs.
  b) Interpret constants and coefficients in the context of the problem.

2.07   Use equations with radical expressions to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the degree, constants, and coefficients in the context of the problem.

2.08   Use equations and inequalities with absolute value to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the constants and coefficients in the context of the problem.

2.09   Use the equations of parabolas and circles to model and solve problems; justify results.
  a) Solve using tables, graphs, and algebraic properties.
  b) Interpret the constants and coefficients in the context of the problem.

2.10   Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

Technical Mathematics 1

Technical Mathematics 1 continues students' study of algebra and geometry, building upon middle school and Algebra I topics. Problem solving, measurement, special relationships in right triangles, transformations, and geometric applications of algebra are the topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Apply geometric properties and relationships to solve problems.
- Use tables, formulas and algebraic expressions to model and solve problems.
- Define and use linear functions to model and solve problems.
- Operate with matrices to model and solve problems.

Strands: Number and Operations, Geometry and Measurement, Algebra

COMPETENCY GOAL 1: The learner will apply various strategies to solve problems.

Objective
1.01   Apply various techniques and strategies to solve problems.
- a) Select or create an appropriate graphical display for a given set of data.
- b) Identify and represent patterns using appropriate algebraic notation.
- c) Select and apply appropriate formulas.
- d) Choose or create appropriate representations of two- and three-dimensional figures.

COMPETENCY GOAL 2: The learner will measure and apply geometric concepts to solve problems.

Objectives
2.01   Select and use appropriate tools to measure two- and three-dimensional figures; interpret and communicate results with regard to precision.
2.02   Interpret and construct maps and scale drawings to solve problems.

2.03   Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.

2.04   Use the trigonometric ratios to model and solve problems involving right triangles.

COMPETENCY GOAL 3: The learner will describe the transformation of polygons in the coordinate plane algebraically.

Objectives

3.01   Apply algebraic and trigonometric concepts to confirm properties of geometric figures in the coordinate plane.

3.02   Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

3.03   Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

Technical Mathematics 2

Technical Mathematics 2 continues students' study of algebra and geometry, building upon middle and high school topics. Geometry, functions, and statistical methods for estimation and prediction are the topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Apply geometric properties and relationships to solve problems.
- Use tables, formulas and algebraic expressions to model and solve problems.
- Identify, graph, and use exponential and quadratic functions to solve problems.
- Create linear models, for a set of data, to solve problems.

Strands: Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will use properties of geometric figures to solve problems.

Objectives
1.01   Apply the properties and definitions of lines and angles to model and solve problems.
1.02   Apply the properties and definitions of plane figures to model and solve problems:
    a) Triangles.
    b) Quadrilaterals.
    c) Other polygons.
    d) Circles.

COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.

Objectives

2.01   Use quadratic equations to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret the constants and coefficients in the context of the problem.

2.02   Use exponential functions to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret the constants, coefficients, and bases in the context of the problem.

2.03   Create, interpret, and analyze best-fit models of linear, exponential, and quadratic functions to solve problems.
   a) Interpret the constants, coefficients, and bases in the context of the data.
   b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

2.04   Use systems of equations and inequalities to model and solve problems using tables, graphs, matrix operations, and algebraic properties; justify results.

Advanced Functions and Modeling

Advanced Functions and Modeling provides students an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, and exponential functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

Strands: Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will analyze data and apply probability concepts to solve problems.

Objectives

1.01   Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems.
   a) Interpret the constants, coefficients, and bases in the context of the data.
   b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions and make predictions.

1.02   Summarize and analyze univariate data to solve problems.
   a) Apply and compare methods of data collection.
   b) Apply statistical principles and methods in sample surveys.

c) Determine measures of central tendency and spread.

d) Recognize, define, and use the normal distribution curve.

e) Interpret graphical displays of univariate data.

f) Compare distributions of univariate data.

1.03 Use theoretical and experimental probability to model and solve problems.

a) Use addition and multiplication principles.

b) Calculate and apply permutations and combinations.

c) Create and use simulations for probability models.

d) Find expected values and determine fairness.

e) Identify and use discrete random variables to solve problems.

f) Apply the Binomial Theorem.

COMPETENCY GOAL 2: The learner will use functions to solve problems.

Objectives

2.01 Use logarithmic (common, natural) functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.

b) Interpret the constants, coefficients, and bases in the context of the problem.

2.02 Use piecewise-defined functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.

b) Interpret the constants, coefficients, and bases in the context of the problem.

2.03 Use power functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.

b) Interpret the constants, coefficients, and bases in the context of the problem.

2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.

b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.

c) Develop and use the law of sines and the law of cosines.

2.05 Use recursively-defined functions to model and solve problems.

a) Find the sum of a finite sequence.

b) Find the sum of an infinite sequence.

c) Determine whether a given series converges or diverges.

d) Translate between recursive and explicit representations.

Discrete Mathematics

Discrete Mathematics introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear and exponential functions to model and solve problems.
- Operate with matrices to model and solve problems.
- Define complex numbers and perform basic operations with them.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will use matrices and graphs to model relationships and solve problems.

Objectives
1.01   Use matrices to model and solve problems.
   a) Display and interpret data.
   b) Write and evaluate matrix expressions to solve problems.
1.02   Use graph theory to model relationships and solve problems.

COMPETENCY GOAL 2: The learner will analyze data and apply probability concepts to solve problems.

Objectives
2.01   Describe data to solve problems.
   a) Apply and compare methods of data collection.
   b) Apply statistical principles and methods in sample surveys.
   c) Determine measures of central tendency and spread.
   d) Recognize, define, and use the normal distribution curve.
   e) Interpret graphical displays of data.
   f) Compare distributions of data.

2.02   Use theoretical and experimental probability to model and solve problems.
   a) Use addition and multiplication principles.
   b) Calculate and apply permutations and combinations.
   c) Create and use simulations for probability models.
   d) Find expected values and determine fairness.
   e) Identify and use discrete random variables to solve problems.
   f) Apply the Binomial Theorem.

2.03   Model and solve problems involving fair outcomes:
   a) Apportionment.
   b) Election Theory.
   c) Voting Power.
   d) Fair Division.

COMPETENCY GOAL 3: The learner will describe and use recursively-defined relationships to solve problems.

Objective
3.01   Use recursion to model and solve problems.
   a) Find the sum of a finite sequence.
   b) Find the sum of an infinite sequence.
   c) Determine whether a given series converges or diverges.
   d) Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas.
   e) Verify an explicit definition with inductive proof.

Pre-Calculus

Pre-Calculus provides students an honors-level study of trigonometry, advanced functions, analytic geometry, and data analysis in preparation for calculus. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will describe geometric figures in the coordinate plane algebraically.

Objectives
1.01 Transform relations in two dimensions; describe the results algebraically and geometrically.
1.02 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret the constants and coefficients in the context of the problem.
1.03 Operate with vectors in two dimensions to model and solve problems.

COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.

Objectives

2.01   Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.
   a) Solve using graphs and algebraic properties.
   b) Interpret the constants, coefficients, and bases in the context of the problem.

2.02   Use trigonometric and inverse trigonometric functions to model and solve problems; justify results.
   a) Solve using graphs and algebraic properties.
   b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
   c) Develop and use the law of sines and the law of cosines.

2.03   For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.
   a) Interpret the constants, coefficients, and bases in the context of the data.
   b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.

2.04   Use the composition and inverse of functions to model and solve problems.

2.05   Use polar equations to model and solve problems.
   a) Solve using graphs and algebraic properties.
   b) Interpret the constants and coefficients in the context of the problem.

2.06   Use parametric equations to model and solve problems.

2.07   Use recursively-defined functions to model and solve problems.
   a) Find the sum of a finite sequence.
   b) Find the sum of an infinite sequence.
   c) Determine whether a given series converges or diverges.
   d) Translate between recursive and explicit representations.

2.08   Explore the limit of a function graphically, numerically, and algebraically.

Integrated Mathematics 1

Integrated Mathematics 1 provides students the opportunity to study traditional topics from algebra, geometry, probability, and statistics in a problem-centered, connected approach. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Operate with real numbers to solve problems.
- Use formulas to solve problems.
- Find, identify, and interpret the slope and intercepts of a linear relation.
- Visually determine the line of best fit for a given scatterplot; explain the meaning of the line; and make predictions using the line.
- Collect, organize, analyze, and display data to solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will perform operations with numbers and expressions to solve problems.

Objectives
1.01   Write equivalent forms of algebraic expressions to solve problems.
      a) Apply the laws of exponents.
      b) Operate with polynomials.
      c) Factor polynomials.
1.02   Use algebraic expressions, including iterative and recursive forms, to model and solve problems.

COMPETENCY GOAL 2: The learner will use properties of geometric figures to solve problems.

Objectives
2.01 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.

2.02 Develop and apply properties of solids to solve problems.

COMPETENCY GOAL 3: The learner will analyze data and apply probability concepts to solve problems.

Objectives
3.01 Use graph theory to model relationships and solve problems.

3.02 Use theoretical and experimental probability to model and solve problems.
   a) Use addition and multiplication principles.
   b) Calculate and apply permutations and combinations.
   c) Create and use simulations for probability models.
   d) Find expected values and determine fairness.

3.03 Create linear and exponential models, for sets of data, to solve problems.
   a) Interpret the constants, coefficients, and bases in the context of the data.
   b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.

Objectives
4.01 Use linear functions or inequalities to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret the constants and coefficients in the context of the problem.

4.02 Use exponential functions to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret the constants, coefficients, and bases in the context of the problem.

4.03 Use systems of linear equations or inequalities in two variables to model problems and solve graphically.

Integrated Mathematics 2

Integrated Mathematics 2 continues students' study of topics from algebra, geometry, and statistics in a problem-centered, connected approach. Functions, matrix operations, and algebraic representations of geometric concepts are the principle topics of study. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Create linear and exponential models, for sets of data, to solve problems.
- Use linear expressions to model and solve problems.
- Collect, organize, analyze, and display data to solve problems.
- Apply geometric properties and relationships to solve problems.
- Apply the Pythagorean Theorem to solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will perform operations with numbers, algebraic expressions, and matrices to solve problems.

Objectives
1.01 Write equivalent forms of algebraic expressions to solve problems.
1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03 Model and solve problems using direct variation.
1.04 Operate with matrices to model and solve problems.

COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.

Objectives
2.01   Find the lengths and midpoints of segments to solve problems.
2.02   Use the parallelism or perpendicularity of lines and segments to solve problems.
2.03   Use the trigonometric ratios to model and solve problems.
2.04   Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

COMPETENCY GOAL 3: The learner will collect, organize, and interpret data to solve problems.

Objectives
3.01   Describe data to solve problems.
   a) Apply and compare methods of data collection.
   b) Apply statistical principles and methods in sample surveys.
   c) Determine measures of central tendency and spread.
   d) Recognize, define, and use the normal distribution curve.
   e) Interpret graphical displays of data.
   f) Compare distributions of data.
3.02   Create and use, for sets of data, calculator-generated models of linear, exponential, and quadratic functions to solve problems.
   a) Interpret the constants, coefficients, and bases in the context of the data.
   b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.

Objectives

4.01   Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify steps used.

4.02   Use quadratic functions to model and solve problems; justify results.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret the constants and coefficients in the context of the problem.

4.03   Use power models to solve problems.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret the constants, coefficients, and bases in the context of the problem.

Integrated Mathematics 3

Integrated Mathematics 3 continues students' study of topics from algebra, geometry, and statistics in a problem-centered, connected approach. Functions and the deductive methods of proof with geometric concepts are the principle topics of study. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Use the trigonometric ratios to model and solve problems.
- Apply geometric properties and relationships to solve problems.
- Use systems of linear equations or inequalities to model and solve problems.
- Define and use linear and exponential functions to model and solve problems.

Strands: Number and Operations, Geometry and Measurement, Algebra

COMPETENCY GOAL 1: The learner will perform operations with numbers and algebraic expressions to solve problems.

Objectives
1.01   Write equivalent forms of algebraic expressions to solve problems.
1.02   Use algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03   Simplify and perform operations with rational exponents and logarithms to solve problems.
1.04   Model and solve problems using direct, inverse, combined and joint variation.

COMPETENCY GOAL 2: The learner will use properties of geometric figures to solve problems.

    Objectives

2.01   Use logic and deductive reasoning to draw conclusions and solve problems.

2.02   Apply the properties, definitions, and theorems of angles and lines to solve problems and write proofs.

2.03   Apply the properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs:
    a) Triangles.
    b) Quadrilaterals.
    c) Other polygons.
    d) Circles.

2.04   Use the law of sines and law of cosines to solve problems.

COMPETENCY GOAL 3: The learner will use relations and functions to solve problems.

    Objectives

3.01   Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

3.02   Use quadratic functions and inequalities to model and solve problems; justify results.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret the constants and coefficients in the context of the problem.

3.03   Use rational equations to model and solve problems; justify results.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret the constants and coefficients in the context of the problem.
    c) Identify the asymptotes and intercepts graphically and algebraically.

3.04   Use equations and inequalities with absolute value to model and solve problems; justify results.
    a) Solve using tables, graphs, and algebraic properties.
    b) Interpret the constants and coefficients in the context of the problem.

3.05   Transform functions in two dimensions; describe the results algebraically and geometrically.

Integrated Mathematics 4

Integrated Mathematics 4 provides students an advanced study of trigonometry, functions, analytic geometry, and data analysis with a problem-centered, connected approach in preparation for college-level mathematics. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

Strands: Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will operate with complex numbers and vectors to solve problems.

> **Objectives**
> 1.01   Operate with vectors in two dimensions to model and solve problems.
> 1.02   Define and compute with complex numbers.

COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.

Objectives
2.01   Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.
     a) Solve using tables, graphs, and algebraic properties.
     b) Interpret the constants and coefficients in the context of the problem.
2.02   Estimate the area and volume of continuously varying quantities.

COMPETENCY GOAL 3: The learner will analyze data to solve problems.

Objectives
3.01   Analyze univariate data to solve problems.
     a) Apply and compare methods of data collection.
     b) Apply statistical principles and methods in sample surveys.
     c) Determine measures of central tendency and spread.
     d) Recognize, define, and use the normal distribution curve.
     e) Interpret graphical displays of distributions of univariate data.
     f) Compare distributions of univariate data.
3.02   Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions of bivariate data to solve problems.
     a) Interpret the constants, coefficients, and bases in the context of the data.
     b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.

COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.

Objectives
4.01   Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.
     a) Solve using graphs and algebraic properties.
     b) Interpret the constants, coefficients, and bases in the context of the problem.

4.02   Use recursively-defined functions to model and solve problems.

- a) Find the sum of a finite sequence.
- b) Find the sum of an infinite sequence.
- c) Determine whether a given series converges or diverges.
- d) Translate between recursive and explicit representations.

4.03   Use the composition and inverse of functions to model and solve problems.

4.04   Use trigonometric and inverse trigonometric functions to model and solve problems.

- a) Solve using graphs and algebraic properties.
- b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.

4.05   Use polar equations to model and solve problems.

- a) Solve using graphs and algebraic properties.
- b) Interpret the constants and coefficients in the context of the problem.

4.06   Use parametric equations to model and solve problems.

4.07   Find the rate of change at any point of a function.

Advanced Placement Statistics

> Advanced Placement Statistics introduces students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Students will observe patterns and departures from patterns, decide what and how to measure, produce models using probability and simulation, and confirm models. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
>
> Prerequisites
> - Create and use, for sets of data, best-fit mathematical models of functions to solve problems.
> - Use logic and deductive reasoning to draw conclusions and solve problems.
> - Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
> - Define and use functions to model and solve problems.

Strands: Number and Operations, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will analyze univariate data to solve problems.

Objectives

1.01   Summarize distributions of univariate data by determining and interpreting measures of center, spread, position, boxplots, and effects of changing units on summary measures.

1.02   Analyze distribution of continuous univariate data (both normal and non-normal).

COMPETENCY GOAL 2: The learner will construct and interpret displays of univariate data to solve problems.

Objectives

2.01   Construct and interpret graphical displays of univariate data

2.02   Compare distributions among sets of univariate data.

COMPETENCY GOAL 3: The learner will collect and analyze date to solve problems.

Objectives

3.01   Analyze categorical data.

3.02   Use and compare methods of data collection.

3.03   Apply statistical principles and methods in sample surveys; identify difficulties.

3.04   Apply principles and methods in designed experiments; identify difficulties.

3.05   Apply concepts of probability to solve problems.

3.06   Use normal distributions as a model for distribution.

    a) Investigate the properties of the normal distribution.

    b) Use the table of standard normal distribution (Z).

3.07   Simulate sampling distributions.

3.08   Use simulations to develop an understanding of the Central Limit Theorem and its importance in confidence intervals and tests of significance.

3.09   Recognize, construct and interpret results using confidence intervals in the context of a problem.

3.10   Perform tests of significance and interpret results in the context of a problem.

COMPETENCY GOAL 4: The learner will analyze bivariate data to solve problems.

Objective

4.01   Analyze bivariate data.

    a) Recognize and analyze correlation and linearity.

    b) Determine the least squares regression line.

    c) Create residual plots and identify outliers and influential points to analyze data.

    d) Use logarithmic and power transformations to analyze data.

Advanced Placement Calculus

Advanced Placement Calculus develops the student's understanding of the concepts of calculus (functions, graphs, limits, derivatives and integrals) and provides experience with its methods and applications. The course encourages the geometric, numerical, analytical, and verbal expression of concepts, results, and problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites
- Use circle, trigonometric, and inverse trigonometric functions to solve problems.
- Use the trigonometric ratios and the laws of sines and cosines to solve problems.
- Describe graphically, algebraically and verbally phenomena as functions; identifying independent and dependent quantities, domain, and range.
- Translate among graphic, algebraic, tabular, and verbal representations of relations.
- Use functions (linear, polynomial, exponential, logarithmic, rational, power, piecewise) to model and solve problems.
- Use the composition and inverse of functions to model and solve problems.
- Transform relations in two and three dimensions; describe algebraically and/or geometrically the results.
- Use the conic relations to model and solve problems.
- Write equivalent forms of algebraic expressions.
- Find special points (zeros, intercepts, asymptotes, local maximum, local minimum, etc.) of relations and describe in the context of the problem.

Strands: Number and Operations, Geometry and Measurement, Algebra

COMPETENCY GOAL 1: The learner will demonstrate an understanding of the behavior of functions.

Objectives
1.01   Demonstrate an understanding of limits both local and global.
- a) Calculate limits, including one-sided, using algebra.
- b) Estimate limits from graphs or tables of data.

1.02   Recognize and describe the nature of aberrant behavior caused by asymptotes and unboundedness.
    a) Understand asymptotes in terms of graphical behavior.
    b) Describe asymptotic behavior in terms of limits involving infinity.
    c) Compare relative magnitudes of functions and their rates of change.

1.03   Identify and demonstrate an understanding of continuity of functions.
    a) Develop an intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
    b) Understand continuity in terms of limits.
    c) Develop a geometric understanding of graphs of continuous functions. (Intermediate Value Theorem and Extreme Value Theorem).

COMPETENCY GOAL 2: The learner will use derivatives to solve problems.

Objectives

2.01   Explore and interpret the concept of the derivative graphically, numerically, analytically and verbally.
    a) Interpret derivative as an instantaneous rate of change.
    b) Define derivative as the limit of the difference quotient.
    c) Identify the relationship between differentiability and continuity.

2.02   Apply the concept of the derivative at a point.
    a) Find the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
    b) Find the tangent line to a curve at a point and local linear approximation.
    c) Find the instantaneous rate of change as the limit of average rate of change.
    d) Approximate a rate of change from graphs and tables of values.

2.03   Interpret the derivative as a function.
    a) Identify corresponding characteristics of graphs of f and f'.
    b) Identify relationship between the increasing and decreasing behavior of f and the sign of f'.
    c) Investigate the Mean Value Theorem and its geometric consequences.
    d) Translate between verbal and algebraic descriptions of equations involving derivatives.

2.04 Demonstrate fluency and accuracy in the computation of derivatives.

a) Find the derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.

b) Use the basic rules for the derivative of sums, products, and quotients of functions.

c) Use the chain rule and implicit differentiation.

2.05 Interpret the second derivative.

a) Identify the corresponding characteristics of the graphs of f, f', and f''.

b) Identify the relationship between the concavity of f and the sign of f''.

c) Identify points of inflection as places where concavity changes.

2.06 Apply the derivative in graphing and modeling contexts.

a) Analyze curves, with attention to monotonicity and concavity.

b) Optimize with both absolute (global) and relative (local) extrema.

c) Model rates of change, including related rates problems.

d) Use implicit differentiation to find the derivative of an inverse function.

e) Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

f) Interpret differential equations geometrically via slope fields and the relationship between slope fields and solution curves for differential equations.

COMPETENCY GOAL 3: The learner will use integrals to solve problems.

Objectives

3.01 Explore and interpret the concept of the definite integral.

a) Compute Riemann sums using left, right, and midpoint evaluation points.

b) Find the definite integral as a limit of Riemann sums over equal subdivisions.

c) Find the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

d) Identify basic properties of definite integrals.

3.02 Apply standard techniques of anti-differentiation.
   a) Find anti-derivatives following directly from derivatives of basic functions.
   b) Find anti-derivatives by substitution of variables. (including change of limits for definite integrals).

3.03 Apply and interpret the Fundamental Theorem of Calculus.
   a) Use the Fundamental Theorem to evaluate definite integrals.
   b) Use the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined.

3.04 Define and use appropriate integrals in a variety of applications.
   a) Interpret the integral of a rate of change to give accumulated change.
   b) Find specific anti-derivatives using initial conditions.
   c) Set up and use an approximating Riemann sum or trapezoidal sum and represent its limit as a definite integral.
   d) Find the area of a region.
   e) Find the volume of a solid with known cross sections.
   f) Find the average value of a function.
   g) Find the distance traveled by a particle along a line.
   h) Solve separable differential equations and use them in modeling. In particular, study the equation $y' = ky$ and exponential growth.